ELEMENTARY PHYSICS

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PREFACE

This book presents the central ideas of physics without using any mathematics more advanced than simple algebra. It is especially designed for the one-semester or one-quarter course usually offered to nonscience majors. The main ideas of each chapter are applied to everyday situations, so that the student's interest will be aroused as he is able to relate physics to the world in which he lives. This world unceasingly obeys the laws of physics.

The first chapter introduces the student to the various areas of physical science and the scientific, or inductive, method of reasoning. An optional review of basic mathematics follows, which covers fractions, proportion, algebra, and exponential notation. The chapter on dynamics includes concepts that are fundamental to all of physics: force, mass, work, power, energy, and momentum. Coverage of temperature and heat energy leads to generalizations using the law of conservation of energy and is a basis for the discussion of entropy. One chapter on electrodynamics deals with the effects produced by both motionless and moving charges. Another chapter on the general aspects of wave motion emphasizes its lesson by using the familiar examples of sound and light waves. Discussions of the discovery of the electron, black-body radiation, and the photoelectric effect lead to a study of atomic spectra and structure, radioactivity, and the nucleus of the atom. The final

chapter suggests advances in physics that may occur in the future — possibilities that may be realized within the student's lifetime.

All of the material included in this book probably cannot be covered in a brief course on physics, so the instructor has the liberty of selecting which topics are to be presented. Sections of greater difficulty or lesser importance are starred and can be omitted without loss of continuity. Numerous problems and class discussion questions are given at the end of each chapter, and these should be regarded as an integral part of the text. There are also many worked-out examples, which, if carefully studied, will be very helpful. A comprehensive teacher's manual is available.

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CONTENTS

FIVE Wave Motion 146

SIX Atomic Physics 188

*AN ASTERISK INDICATES SECTIONS THAT ARE MORE DIFFICULT MATHEMATICALLY OR OF LESSER IMPORTANCE. THEY MAY BE OMITTED WITHOUT LOSS OF CONTINUITY.

ELEMENTARY PHYSICS

CHAPTER ONE

INTRODUCTION

1.1 The nature of physics

The first question that the reader of this book probably has in his mind concerns the nature of physics. As a starting point we can say that *physics* is concerned with the properties of inanimate materials and their interactions with the rest of the universe. This is in contrast with *biology*, which deals primarily with living things and their environments. Naturally there are many situations where the two fields overlap. For instance, the circulation of the blood is important to the biologist and yet is also of interest to the physicist as a difficult problem in fluid flow. Many of these

borderline areas are included in the field of knowledge known as *biophysics*. Similarly, *chemistry* and *geology* are usually treated as sciences distinct from physics, even though their fundamental theories are based on physics. Here again we find overlapping areas of knowledge, such as *geophysics* and *physical chemistry*. An outstanding achievement during the first half of this century has been the gradual merger of chemistry, geology, astronomy, and physics into the larger field of *physical science*. During the next few decades we can expect the further joining of physical science with biology into a single science with many subdivisions.

A second question which probably has occurred to the reader of this book deals with the problems and situations which are studied in physics. In the definition of physics in the preceding paragraph the word "property" was used in its most general sense, so as to include almost all physical aspects of any situation. For instance, the physicist may be interested in the weight of a body, the effect of gravity on a body, the temperature of a body, the electrical state of a body, and many other properties which a body might possess. On the other hand, we might want to know the structure and composition of a body beginning with the theory of atoms and molecules and proceeding to the large-scale aspects of the body. Or we might deal with stars, planets, and satellites so that we would finally be concerned with the structure, development, and origin of the universe. Another important area of interest is the properties of waves, such as light and radio waves. Many more examples could be given, but the ones above should indicate typical branches of physics. In this book we will take up the central ideas of physics without spending much time on applications or detailed areas which are found in courses in chemistry, engineering, and advanced physics.

1.2 The scientific method

An excellent example of a much-abused term is "scientific method." To many persons not acquainted with science this is some magical method by which scientists find out the truth, and all that is necessary is to apply this method properly in a given

situation. While there is such a thing as the scientific method, as we will try to explain below, it is neither magical nor sure of success. However, it should be pointed out here that the rise in importance of science and technology can be said to begin with the first applications of the scientific method in the early part of the seventeenth century, so that it has been of immense value to mankind even though it is not infallible.

Just what is the scientific method? Usually the procedure is about as follows: First, one observes something in nature and looks for regularities. For instance, we notice that the sun rises in the east at approximately the same time every morning. We notice also that the sun, moon, and stars all seem to move in circular paths about the earth. Having thus observed a regularity in nature, our next job is to construct a theory or hypothesis to explain these facts. If we lived a few centuries ago, we would assume as the simplest explanation that all objects in the heavens really do move about the earth in circular paths. Setting up a theory is then the second step in the scientific method. The third and crucial step is to make a prediction based on our theory and test it by experiment. For instance, we might predict that sunrise would occur at the same time every morning. Within a few days we would find that this is not quite correct. The next step is then to modify our theory so as to agree with this new piece of evidence. Thus, in a typical case we learn facts by experiment, modify our theory to agree with new facts, then test the new theory. This is continued until satisfactory agreement between theory and experiment is found. At no time, however, can we say that the theory is true in any exact or mathematical sense. At best we can say that our theory is sufficient to describe the currently known facts. Later discoveries or more accurate measurements may require changes in the theory or even a completely different theory. The history of science is full of examples of theories which were apparently successful for years but had to be abandoned or greatly changed when new evidence was found experimentally.

To sum up, the *scientific or inductive method* is based primarily on facts obtained from experiments. Theories or hypotheses are

then established which agree with (explain) these facts. Predictions of the theory are then tested experimentally, and if the results agree with the theory, we gain confidence in it. If not, we change the theory. The reader should notice that no number of successes for a theory ever proves it true, while a single failure of the theory immediately proves it wrong. Of course, in many cases, a minor change in a theory will enable it to agree with the new fact. After a theory has been used successfully for a long time in predicting the results of many experiments, it then is given the status of a *law*. However, many theories dignified by being called laws have later been found to be incorrect in some details and have been replaced by better theories. No theory is accepted merely because it seems plausible or reasonable. The test of any theory is its agreement with the experimental facts, although a choice between two theories which are equally good at explaining observations is usually made on the basis of simplicity and reasonableness.

At this point a contrast between the deductive and inductive methods might be useful to the reader. A good example of the deductive method is plane geometry as originally proposed by Euclid. Here we begin with certain definitions and propositions (axioms). Deductions from these statements are made using the rules of logic. The conclusions then are "true" in the sense that they follow logically from the original statements of the theory. Of course, if the definitions and propositions do not agree with the "real" world, then the deductions from them will not be found to be correct experimentally. On the other hand, in the inductive method we begin by making observations. From these facts we try to work back to find out the proper definitions and propositions which will then lead to a prediction of the experimental observations. The fault in the deductive method is that we are never sure that the original statements agree with the real world, so that deductions may not necessarily be physically correct even though they are correct logically. The difficulty with the inductive method is that our theory may be merely sufficient to explain the limited number of experimental facts we have to the accuracy involved. A single contradiction of theory

derived inductively destroys it. In this respect, the inductive or scientific method can never arrive at the truth, but can at best give us today's closest approximation, subject to revision—possibly tomorrow. Some examples are given below of failures of the inductive method.

As a ludicrous example of the failure of the inductive method, consider the possible behavior of illiterate savages during the eclipse of the sun. They might decide that an immediate sacrifice of a maiden would make the sun return. This theory would seem to be completely correct, as the sun would reappear from behind the moon in a matter of minutes. However, as a test of the theory, some wise member of the tribe might propose either that they do nothing (a negative test) or sacrifice a lamb instead. In either case, of course, the sun would still reappear very quickly, thus destroying the original theory.

You might hear a radio announcer say the numbers 10, 20, 30, 40, 50. If asked for the next number in the series you might guess 60. While this would probably be correct in most cases, the next number might be 40, as the announcer could be describing the runback of a kickoff in a football game. Again the original theory was sufficient to describe the first five numbers heard, but might not be correct in predicting the sixth number. Only experiment (listening some more to the radio) could tell if the original theory was correct or not. Even then there would be no certainty that the seventh number would be predicted correctly.

To summarize, in the scientific or inductive method, the first step is to observe systematically. A theory is then contrived to fit (explain) these facts. Predictions of the theory are then tested experimentally. If the theory agrees with these new facts, one gets more confidence in it and continues to use it until it fails. If the theory does not agree with new facts, then it is either modified or replaced by a new and generally broader theory. After a theory has been successful for many years in accounting for a wide range of facts, it is often honored by the title of "law." Nevertheless, whether we are dealing with a new theory or an ancient law, a single exception forces either modification or

abandonment of it. Thus, although the scientific method and its applications have been very useful in advancing our knowledge and use of nature, by its very nature it cannot lead to absolute truth, if such a thing exists.

*1.3 Review of fractions and simple proportion

Consider the fraction $\frac{6}{8}$. We can simplify this fraction by dividing the numerator (6) and the denominator (8) each by 2, obtaining the equivalent fraction $\frac{3}{4}$. Similarly, the value of the fraction would be unchanged if we were to multiply the numerator and the denominator each by the same factor. If this factor is 3, for instance, the equivalent fraction would be $\frac{18}{24}$. Depending on the situation, we may benefit by changing a fraction to an equivalent form, as described above. In general terms, if we begin with the fraction a/b, then the equivalent fraction is na/nb for any value of n, positive or negative, since $n/n = 1$.

Let us now consider adding $\frac{3}{4}$ to $\frac{1}{8}$. Since fourths and eighths are as different as oranges and apples, we must first arrange things so that both fractions have the same denominator. We do this by multiplying the numerator and the denominator of the first fraction by 2. We find then that

$$\tfrac{3}{4} + \tfrac{1}{8} = \tfrac{6}{8} + \tfrac{1}{8} = \tfrac{7}{8}$$

As a more difficult case, let us add $\frac{2}{3}$ and $\frac{3}{7}$. Again we must make the denominators of the fractions the same before we can add them. If we multiply the first fraction by $\frac{7}{7}$ and the second by $\frac{3}{3}$, we find

$$\tfrac{2}{3} + \tfrac{3}{7} = \tfrac{7}{7}(\tfrac{2}{3}) + \tfrac{3}{3}(\tfrac{3}{7}) = \tfrac{14}{21} + \tfrac{9}{21} = \tfrac{23}{21}$$

In the example just above we could just as well have written that

$$\frac{2}{3} + \frac{3}{7} = \frac{2 \times 7 + 3 \times 3}{3 \times 7} = \frac{14 + 9}{21} = \frac{23}{21}$$

In general, if we wish to add the fractions a/b and c/d, we proceed in the same way. Thus we write:

$$\frac{a}{b}+\frac{c}{d}=\frac{d\,a}{d\,b}+\frac{b\,c}{b\,d}=\frac{da+bc}{bd}$$

The final fraction may then possibly be simplified as described in the paragraph above. Subtraction of fractions is basically the same as addition, except that now we must pay attention to the algebraic signs of the numbers. Consider the following example:

$$\tfrac{2}{3}-\tfrac{3}{7}=\tfrac{14}{21}-\tfrac{9}{21}=\tfrac{5}{21}$$

In the example given above in terms of letters, one or more of the letters a, b, c, d may have a negative sign which must be taken into account in computing the value of the final fraction.

Suppose now that we must combine three or more fractions. We will treat the case of three fractions, but it is easy to extend the ideas to cover cases involving four or more fractions. As an example, consider adding $\frac{2}{7}$, $\frac{1}{3}$, and $\frac{3}{5}$. The first step is to make all three fractions have the same denominator, which can be done by multiplying the numerator and denominator of each fraction by the product of the denominators of the *other* fractions which are to be added. In this case we have:

$$\frac{2}{7}+\frac{1}{3}+\frac{3}{5}=\left(\frac{3\times 5}{3\times 5}\right)\left(\frac{2}{7}\right)+\left(\frac{5\times 7}{5\times 7}\right)\left(\frac{1}{3}\right)+\left(\frac{7\times 3}{7\times 3}\right)\left(\frac{3}{5}\right)$$

$$=\frac{30}{105}+\frac{35}{105}+\frac{63}{105}=\frac{128}{105}$$

If one or more of the fractions is to be subtracted, a minus sign will appear in place of the plus sign for each such negative fraction. After all fractions have been changed so as to have the same denominator, they are then added with due attention to the sign of each numerator. Often the resulting fraction can be put in simpler form by dividing its numerator and denominator by some common factor.

Decimal fractions are a special but very common type of fraction for which a different notation is used. For instance, in place of 5/10 it is customary to write 0.5. Similarly 0.012 stands for 0.12/10 which is the equivalent of 1.2/100 or 12/1000. The rule is simple: count the number of digits to the right of the decimal point, including zeros, to find the number of zeros in the denominator. The numerator is then the number to the right of the decimal point, with the decimal point left out. For instance, $0.348 = 348/1000, 0.0204 = 204/10000,$ and $23/1000 = 0.023$. Decimal fractions are particularly useful since it is so easy to multiply and divide by ten or a multiple of ten. The first digit to the right of the decimal point gives the number of tenths, the next digit to the right gives the number of hundredths, the third digit to the right of the decimal point gives the number of thousandths, and so forth. Consider adding 0.135 and 0.203. To get this sum we write

$$\begin{array}{r} 0.135 \\ +0.203 \\ \hline 0.338 \end{array}$$

Suppose that in adding a part of a decimal fraction we get, for instance, 12/100. Since 12/100 is equivalent to $1/10 + 2/100$, we simply carry one unit to the tenths addition. As an example consider adding 0.386 to 0.035. We have then:

$$\begin{array}{r} 0.386 \\ +0.035 \\ \hline 0.421 \end{array}$$

In the example above, we first found 11/1000, so that we carried one into the hundredths column. Then we got 12/100, so that we carried one into the tenths column. The extension of these ideas to the addition or subtraction of more than two decimal fractions is easy and will be left for exercises at the end of this chapter.

Consider the identity $\frac{2}{3} = \frac{4}{6}$. In words we can say that 2 is to 3 as 4 is to 6. This is called a *simple proportion*. If a recipe calls for $\frac{1}{2}$ cup of flour and we wish to triple the recipe, we must

use $3 \times (\frac{1}{2}) = \frac{3}{2} = 1\frac{1}{2}$ cups of flour. This is an example of a simple proportion which we can calculate very easily. More generally, a simple proportion can be written in the form $a/b = c/d$. Suppose that all of the letters had known values except c. Multiplying each side of the equation by d, we obtain $c = da/b$, from which we can compute c. Alternatively, suppose we knew all of the quantities except b. If we multiply both sides of the equation by bd/c, we get $b = da/c$. Thus we see that in any simple proportion we can compute the value of one factor if the values of the other three factors are known. Proportions occur very frequently in science, so you should master the principles used in solving them. Examples are given below in the text and among the problems at the end of this chapter.

1. $2/3 = n/6$. Multiplying both sides of this equation by 6, we find $n = 6 \times (2/3) = 12/3 = 4$.

2. $3/5 = 10/n$. When we multiply each side of this equation by n, we obtain $(3/5) \times n = 10$. If we now multiply each side of the last equation by $5/3$, we get $n = (5/3) \times 10 = 50/3 = 16\frac{2}{3}$.

3. $2/7 = (n + 3)/28$. First we multiply each side of this equation by 28, finding $28 \times (2/7) = n + 3$. We now have that $8 = n + 3$. After we subtract 3 from each side of the last equation, we find $n = 5$.

*1.4 Review of algebra

The three examples at the end of the preceding section are all simple cases of algebraic equations, since in each case we could solve for an unknown quantity symbolized by a letter in terms of known quantities. Consider the simple equation $3n = 8$. If we divide each side of this equation by 3, we find $n = \frac{8}{3} = 2.67$. A slightly more difficult equation is $2n + 4 = 9$. After we subtract 4 from each side of this equation, we get $2n = 9 - 4 = 5$.

Instead of saying that we have subtracted 4 from each side of the equation above, we could just as well have said that we can move $+4$ from the left side of the equation if we write it as -4 on the right side of the equation. When we now divide each side of the preceding equation by 2, we find $n = \frac{5}{2} = 2.5$. As a third example, let us consider the equation $10 + 3n = 6$. First we subtract 10 from each side of this equation, so that we obtain $3n = 6 - 10 = -4$. (Subtracting 10 from each side of the equation is the same as moving $+10$ to the right side of the equation and writing it as -10.) Thus, $n = -\frac{4}{3} = -1.33\frac{1}{3}$.

The general scheme in solving equations involving one unknown quantity is first to add or subtract terms on both sides of the equation in such a way that the term involving the unknown appears by itself. Then the value of the unknown is found by dividing both sides of the new equation by the coefficient (multiplier appearing in front of a factor) of the unknown. The resulting value for the unknown may be either positive or negative, depending on the problem. The general case is the equation $an + b = c$. We first write $an = c - b$. Then we divide by a, obtaining as our solution $n = (c - b)/a$. Note that in this example the letters, a, b, and c may stand for either positive or negative numbers.

In many physical problems two unknown quantities are related by two equations. Suppose that the unknowns n and m are related by the equations $n + 2m = 8$ and $3n + 4m = 18$. In this case we first solve one of the equations for one of the unknowns. For instance, from the first equation we see that $n = 8 - 2m$. When we substitute this value for n into the second equation, we find $3(8 - 2m) + 4m = 18$. After we multiply together the factors involved in the parenthesis, we get $24 - 6m + 4m = 18$. We can move a quantity from one side of an equation to the other side if we change its sign. When we do this in the equation above we have $24 - 18 = 6m - 4m$. This equation now reduces to $6 = 2m$, from which we see that $m = 3$. Returning to the first equation, we find that $n = 8 - (2 \times 3) = 8 - 6 = 2$. Our problem is now solved. To summarize the procedure, in solving for two unknowns which are related by two

equations, we first solve one of the two equations for one of the unknowns. We then substitute this value of one unknown into the second equation. This reduces the second equation to one involving only a single unknown, which we can now solve for. After we have determined one of the unknowns, we return to the first equation to calculate the other unknown.

1.5 Exponential notation

Let us look at the identities $3 \times 3 = 9$ and $3 \times 3 \times 3 = 27$. In the first case the number 3 occurs twice in the multiplication, while in the second, 3 occurs three times. In order to simplify writing repetitive expressions, it is customary to write $3 \times 3 = 3^2$ and $3 \times 3 \times 3 = 3^3$. These are examples of *exponential notation*. In general, if we wish to multiply the number n by itself m times we write n^m. If m is 1, we mean the number n itself. Thus, $8^1 = 8$ and $n^1 = n$. If m equals zero, this means that the number n is not a factor at all. For this reason, we define 8^0 as 1 and n^0 as 1, regardless of the value of n. Consider now the following identity:

$$\frac{3 \times 3 \times 3}{3 \times 3} = 3$$

In exponential notation the identity above would be written:

$$\frac{3^3}{3^2} = 3^{(3-2)} = 3^1 = 3$$

From this example we see that a number in the denominator of a fraction can be considered as having a negative exponent. Thus, the preceding example could have been written in the form:

$$3^3 \times 3^{-2} = 3^{(3-2)} = 3^1$$

In general we find it useful to mean by n^{-m} the equivalent expression $1/n^m$. In multiplying together powers of the same number we then simply add their exponents, paying due regard to sign

if some of the powers are negative, meaning that they occur in the denominator. As an example, $10^{-2} = 1/10^2 = 0.01$. Similarly, $10^3 = 1000$, and $10^3 \times 10^{-2} = 10^1 = 10$.

The foregoing furnishes the basis for the *power-of-ten notation.* Powers of ten are factored from each number, so that the numerical part left over is between 1 and 10. Thus, $3167 = 3.167 \times 10^3$, and $0.0123 = 1.23 \times 10^{-2}$. In either case we count right or left from the decimal point until we reach a number between 1 and 10. If we count to the left, the power of ten is positive (3 in the example above), while if we count to the right, the power of ten is negative (2 in the example above). One useful feature of this notation is the expression of very large or very small numbers. For instance, instead of writing

$$602300000000000000000000$$

we can write 6.023×10^{23}. Similarly, instead of writing

$$0.000000000000000000000000000000911$$

we can write 9.11×10^{-31}. The other advantage of this notation is that in multiplication and division we can combine the powers of ten very easily by addition and subtraction as explained in the previous paragraph. Here are some examples of this procedure.

Suppose that we would like to multiply 3785 by 163. We can then write:

$$\begin{aligned} 3785 \times 163 &= 3.785 \times 10^3 \times 1.63 \times 10^2 \\ &= (3.785 \times 1.63) \times 10^{3+2} \\ &= 6.17 \times 10^5 \qquad \text{(approximately)} \end{aligned}$$

Next, consider multiplying 450 by 0.12. We can write this in the form:

$$\begin{aligned} 450 \times 0.12 &= 4.50 \times 10^2 \times 1.2 \times 10^{-1} \\ &= 5.40 \times 10^{2-1} \\ &= 5.40 \times 10^1 \\ &= 54 \end{aligned}$$

In the same way we can use the power-of-ten notation where

division is involved. Thus, if we wish to divide 4500 by 30,000 we can write:

$$\frac{4500}{30{,}000} = \frac{4.5 \times 10^3}{3 \times 10^4} = 1.5 \times 10^{3-4} = 1.5 \times 10^{-1} = 0.15$$

A somewhat more difficult problem might involve several numbers. Consider the following example:

$$\frac{24 \times 500}{15 \times 80} = \frac{2.4 \times 10^1 \times 5 \times 10^2}{1.5 \times 10^1 \times 8 \times 10^1} = \frac{12 \times 10^3}{12 \times 10^2} = 10^1 = 10$$

Additional examples are given in the exercises at the end of this chapter and should be worked out by the reader.

While the value of the power-of-ten notation is mainly in its use in expressing large and small numbers and in making arithmetic easier, we can also show most clearly in this notation the estimated accuracy of a number. When we say that we have a dozen oranges, we mean that we have exactly twelve oranges. Mathematically, we might therefore say that we have 12.0000... oranges. In the preceding sentence, the string of dots means that we know the quantity exactly. On the other hand, if we measure the length of a table with a yardstick and say that the length of the table is six feet, we do not claim that the length is exactly six feet. In this case, we might only be able to read the yardstick within a hundredth of a foot, so more carefully we would say that the length of the table is (6 ± 0.01) feet. In the measurement of the size of a cloud, the uncertainty in the measurement would be much larger. In all physical measurements the estimated error can be given as shown above.

If we now consider again the example of the length of the table in the preceding paragraph, another way of showing our estimate of the precision of our measurement is to say that the length is 6.00 feet. By this statement we mean that we think that the length lies between 5.99 feet and 6.01 feet. By convention, unless otherwise stated, we imply an uncertainty of one unit in the last figure quoted. In this example, the value is quoted to three *significant figures*, and the uncertainty occurs in the last significant figure.

Suppose now that I say that the speed of light is 186,000 miles per hour. Do I mean that the speed of light lies between 185,999 and 186,001 miles per hour? Certainly not. However, if I write for the speed of light 1.86×10^5 miles per hour, I now do mean that this value lies between 1.85×10^5 and 1.87×10^5 miles per hour. Thus, by using the power-of-ten notation I can show exactly the number of significant figures to which the value is being quoted.

If I survey a plot for a football field and estimate that my error is probably no more than $\frac{1}{3}$ of an inch (approximately 0.01 yards), I could write that the length of the field was 1.0000×10^2 yards. This statement implies that the uncertainty in the measurement is $0.0001 \times 10^2 = 0.01$ yards. While in this case I could have quoted the length of the field as (100 ± 0.01) yards, when very small or large numbers are involved the use of the power-of-ten notation is clearly much more convenient and clear.

The value of a certain quantity is given as 5.230×10^{10}. In this case the zero is a significant figure. Thus, the probable value of this quantity lies between 5.229×10^{10} and 5.231×10^{10}. Similarly, stating that the value of the quantity is 5.2300×10^{10} implies that the uncertainty is in the second zero, since the value of the quantity was given to five significant figures. In this example we see again that the use of power-of-ten notation gives a precise idea of the accuracy of the measurement, while writing the value as 52,300,000,000 would not.

SUMMARY

In this chapter we began by describing briefly the various areas of knowledge which are of concern to scientists, such as physics, chemistry, and biology. The inter-relations between these sciences were also discussed. This led us to consider the scientific or inductive method which is used in all of the sciences. Typically, the scientist makes some observations and then proposes a theory to explain them. He then tests his theory by seeing if its predictions agree with the results of experiments. As long as the theory continues to be successful it will be

used, but if it fails to agree with experimental evidence it will be modified or discarded.

Since many of the readers of this book do not have occasion to use mathematics very often, the branches of mathematics which will be used in this book were reviewed. The addition and subtraction of fractions and their combination into a single fraction or number will be used many times in later work, so this topic was discussed first. Since most of the quantitative problems we will deal with later can be solved by the use of simple proportion, this technique was considered next. Similarly, we will occasionally have use for elementary algebra, so a review of this was presented. Finally, the exponential or power-of-ten notation was discussed because of its very great usefulness in expressing extremely large or small numbers. The student should refer back to these discussions whenever he runs into mathematical difficulties in later chapters.

PROBLEMS

NOTE: Throughout the problems in this book accuracy to three figures, such as 1.63, will be sufficient except where noted.

1 *You observe that the following identities can be obtained by cancelling common digits in the numerator and denominator:* $\frac{16}{64} = \frac{1}{4}$, $\frac{19}{95} = \frac{1}{5}$, $\frac{26}{65} = \frac{2}{5}$. *Construct a hypothesis based on these facts and test it on other fractions of a similar nature, such as* $\frac{14}{42}$, $\frac{12}{24}$, *and so on. Can you find other fractions for which your hypothesis does work? What do you conclude about the validity of your hypothesis?*

ANS.: *Hypothesis is not valid, although* $\frac{49}{98}$ *and others work.*

2 *We observe that* $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, *and so on. Can you find a regularity existing between the differences between these numbers? If so, see if you can derive it algebraically for the general case. Do you believe that the hypothesis you derived from the data given is correct or not?*

3 *Suppose that the following observations were made on some physical quantity: 2.5, 5, 7.5, 10, 15, 17.5, 20, 25. Determine a law which predicts these numbers and use your law to predict other numbers belonging to the same group.*

ANS.: *Best hypothesis is that* $N = 2.5n$, *where n is an integer.*

4 *Given the fractions $\frac{3}{4}$ and $\frac{7}{12}$. Find their sum and difference.*

5 *Given the fractions $\frac{2}{3}$ and $\frac{4}{7}$. Find their sum and difference.*

ANS.: $\frac{26}{21}$; $\frac{2}{21}$.

6 *Find the sum of the fractions $\frac{3}{5}$, $\frac{4}{10}$, and $\frac{2}{6}$. Reduce your result to its simplest form.*

7 *Find the sum of the fractions $\frac{3}{7}$, $\frac{7}{8}$, and $-\frac{4}{9}$.* ANS.: $\frac{433}{504}$.

8 *A recipe calls for one cup of flour and $\frac{1}{3}$ cup of milk. If you use $1\frac{1}{2}$ cups of milk, how many cups of flour should you use?*

9 *On a turnpike you drive 27 miles during the first half-hour. If you continue driving at the same speed, how long will it take you to cover the next 90 miles of the turnpike?* ANS.: $\frac{5}{3}$ *hours.*

10 *If 3 cubic feet of wood weigh 150 pounds, what is the weight of a log of volume 20 cubic feet?*

11 *Solve the equation $2n + 6 = 20$ for n.* ANS.: *7.*

12 *Solve the equation $3n - 8 = 5$ for n.*

13 *Solve the equation $n/4 + 2 = 3$ for n.* ANS.: *4.*

14 *Solve the equation $n/5 + 8 = 5$ for n.*

15 *Write the number 0.045 as a common fraction and in the power-of-ten notation.* ANS.: $45/1000 = 4.5 \times 10^{-2}$.

16 *Write 135/1000 as a decimal fraction and in the power-of-ten notation.*

17 *Write 6.78×10^{-4} as a common fraction and as a decimal fraction.*
ANS.: $678/1000000 = 0.000678$.

18 *Write 3×10^8 out in full.*

19 *Write 4.8×10^{-10} as a decimal fraction.* ANS.: *0.00000000048.*

20 *Write 186,000 in the power-of-ten notation.*

21 *Write 0.00000000000000000016 in the power-of-ten notation.*
ANS.: 1.6×10^{-19}.

22 *Solve the following two equations for n and m:*

$$n + 3m = 11 \qquad 2n + 4m = 16$$

23 *Solve the following two equations for n and m:*

$$3n + 4m = 9 \qquad 2n - m/6 = 1/3$$

ANS.: $n = \frac{1}{3}$; $m = 2$.

24 *Given the fractions* $\frac{2}{5}$ *and* $\frac{3}{7}$. *Find their sum and difference.*

25 *Given the fractions* $\frac{13}{11}$ *and* $\frac{15}{8}$. *Find their sum and difference.*

ANS.: $\frac{269}{88}$, $-\frac{61}{88}$.

26 *Find the sum of the fractions* $\frac{2}{5}$, $\frac{11}{9}$, *and* $-\frac{2}{3}$.

27 *Find the sum of the fractions* $\frac{13}{10}$, $\frac{2}{5}$, *and* $-\frac{6}{20}$. ANS.: $\frac{13}{10}$.

28 *For an outboard motor you are supposed to add one pint of oil to each gallon of gasoline. If you have two quarts of oil available, how much gasoline should you put in the tank?*

29 *You have $1200 in a savings account at an annual interest of 4 percent. How much interest do you earn per month?*

ANS.: *$4 per month.*

30 *It takes one ounce of syrup with 4 ounces of soda water to make a drink. How much of each do you need to make 20 drinks?*

31 *If a man weighs 180 pounds and this is made up of material weighing on the average 80 pounds per cubic foot, what is the volume of the man in cubic feet?* ANS.: *2.25 cubic feet.*

32 *If a block of stone weighs 200 pounds and has a volume of 2.5 cubic feet, what is the density of this stone in pounds per cubic foot?*

33 *Solve the equation* $3n + 5 = 20$ *for n.* ANS.: 5.

34 *Solve the equation* $4n - 6 = 18$ *for n.*

35 *Solve the equation* $2n - 3 = 8$ *for n.* ANS.: $\frac{11}{2}$.

36 *Solve the equation* $3(n + 6) = 12$ *for n.*

37 *Solve the equation* $5(2n - 4) = 17$ *for n.* ANS.: 3.7.

38 *Solve the equation* $n/3 + 3/4 = 15/4$ *for n.*

39 *Write the number 0.13 as a common fraction and in the power-of-ten notation.* ANS.: $\frac{13}{100}$; 1.3×10^{-1}.

40 *Write the number 0.25 as a common fraction, and in the power-of-ten notation.*

41 *Write* 2×10^{-3} *as a common fraction and as a decimal fraction.*
ANS.: $2/1000$; 0.002.

42 *Multiply* 3×10^{-4} *by* 5×10^{5}.

43 *Divide* 4×10^{5} *by* 3×10^{3}. ANS.: 133.

44 *Multiply* 4.5×10^{3} *by* 2.2×10^{2}.

45 *Divide* 5.2×10^{5} *by* 1.3×10^{-2}. ANS.: 4.0×10^{7}.

46 *Write 0.0000000684 in the power-of-ten notation.*

47 *Write 93,700,000 in the power-of-ten notation.* ANS.: 9.37×10^{7}.

48 *Solve the following equations for n and m:*

$$2n + 3m = 7 \qquad 3n - m = 1$$

49 *Solve the following equations for n and m:*

$$\tfrac{3}{5}n + \tfrac{2}{3}m = 5 \qquad n + m = 7$$

ANS.: $n = -5$; $m = 12$.

50 *Solve the following equations for n and m:*

$$2n + 3m = 7 \qquad 6n + 9m = 21$$

DISCUSSION QUESTIONS

1 *Is the problem of correcting eyesight deficiencies primarily a problem for the biologist or the physicist?*

2 *You are asked to decide safe limits for testing of nuclear bombs. What specialists would you want on your team and what sorts of questions would you ask each of them?*

3 *You are asked to determine if a certain drug is effective in preventing air-sickness. Would you give it to the pilot, the stewardess, or the passengers? Guess the likely results and describe how you would interpret them.*

4 *As you drive through a city on a main avenue, you notice that the names of the cross-streets are Danville, Evelyn, and Francis. What would you conclude from this observation?*

5 *Chicago is served by many airlines and railroads. From this would you conclude that Chicago was built so that it would be near these facilities?*

6 *What odds would you give that the sun will rise tomorrow? Also, what odds would you give that tomorrow will be a clear, sunshiny day? How would you justify each of your bets?*

7 *A* googol *is defined as one followed by a hundred zeros. How would this be written in the power-of-ten notation? Write out a googol in full and see how long it takes you.*

8 *See the preceding question. A* googolplex *is defined as a googol of googols—that is, a googol to the googol power. Write the expression for a googolplex in the power-of-ten notation. Write out a googolplex in full and see how long it takes you.*

9 *Discuss the following statements and conclusion: Men are human. All women are human. Therefore, all humans are women.*

10 *In playing cards you find that the first four cards dealt to you are all aces. What theory would you set up to explain this and what would you do to test it?*

11 *You notice a group of brothers and sisters playing together. They all have red hair. What hypothesis would you set up and how would you test it?*

12 *See question 10 above. All four of the cards are aces of spades. Again, what theory occurs to you and how would you test it?*

13 *What chicanery took place in the following algebraic steps:*

Given	$x = 1$
Multiply by x	$x^2 = x$
Subtract 1	$x^2 - 1 = x - 1$
Factor	$(x + 1)(x - 1) = x - 1$
Cancel $(x - 1)$	$x + 1 = 0$
Thus	$x = -1$

14 *A number of presidents of the United States elected during years divisible by four have died in office. What theory do you propose to account for these facts?*

15 *The New York Yankees baseball team has often failed to win the American League pennant or the World Series in years divisible by four while winning many other years. What hypothesis do you suggest to account for these facts?*

CHAPTER TWO

DYNAMICS

2.1 Standards and units

In describing things quantitatively we begin with standards and units of the quantity which we wish to describe. While the information that a town is "not far away" may be helpful, we would much rather be told that it is about five miles away, or better yet, that it is 5.3 miles away. Thus, the first step is to set up standards and related units for each quantity which we would like to discuss in an objective way. For instance, the standard of the monetary system in the United States is the dollar. As units we have multiples, such as five and ten dollars, and submultiples, such as the

dime ($0.10) and the cent ($0.01). It should be noted that the dollar is defined quite arbitrarily, and the related units are chosen for their convenient magnitudes, which are often related to the standard by powers of ten. The same thing is done in science.

Until the end of the eighteenth century each country had its own set of units, which usually had no relation to units used in other countries. A unit of length, for instance, the inch, was equal to the length of a part of the king's finger. Another unit of length was the fathom, which was the distance between a man's fingertips when his arms were outstretched. Furthermore, relations among the units were not simple. In the English-speaking countries we still use the relations 1 yard = 3 feet = 36 inches. At the time of the French Revolution (1789) an attempt was made to introduce rational and permanent standards as well as to derive the related units from the standards by various powers of ten. This *metric system* was gradually adopted by scientists and is now used throughout most of the world. This is the system which will be chiefly used in this book. For the convenience of the reader, references will be made to the British system of units, which is still in common use in the English-speaking countries.

The standard of length established by the French is called the *meter* from Greek, Latin, and Anglo-Saxon words meaning "measure." Originally the meter was supposed to be 1/40,000,000 of the earth's circumference of a great circle passing through Paris. Thus, the earth itself could always be used to redetermine the length of the meter. However, the best knowledge of the earth's size available at the time was used to indicate the length of the meter as the distance between two scratches on a platinum-iridium bar. When later measurements of the size of the earth showed that the original distance was slightly incorrect, the meter was redefined as being *exactly* the distance marked on the bar. The original meter bar is kept just outside of Paris at Sèvres, France. Other laboratories, such as the U.S. Bureau of Standards, have copies of this meter bar, which are compared with the original bar from time to time. Recently, for experimental reasons the meter has again been redefined, this time in terms of certain

wavelengths of light emitted by krypton. This will not concern us in this book.

Since the meter is a bit over three feet in length, it is not a convenient size for some measurements. In the metric system, multiples and submultiples of the meter using various powers of ten are used as units. Prefixes derived from the Greek are used to designate multiples, as shown in the following table:

deka	10	1 dekameter = 10 meters
hecto	100	1 hectometer = 100 meters
kilo	1000	1 kilometer = 1000 meters
mega	10^6	1 megameter = 10^6 meters
giga	10^9	1 gigameter = 10^9 meters

Similarly, prefixes derived from the Latin are used to indicate submultiples, as shown below:

deci	$\frac{1}{10}$	1 decimeter = $\frac{1}{10}$ meter
centi	$\frac{1}{100}$	1 centimeter = $\frac{1}{100}$ meter
milli	$\frac{1}{1000}$	1 millimeter = $\frac{1}{1000}$ meter
micro	10^{-6}	1 micron = 10^{-6} meter
nano	10^{-9}	1 nanometer = 10^{-9} meter
pico	10^{-12}	1 picometer = 10^{-12} meter

Of the units shown above, the only units of length in common use are the kilometer, meter, centimeter, millimeter, and micron. However, the same prefixes are used to designate multiples and submultiples of other scientific quantities. All of the prefixes listed above are used in one area or another of physics.

The archaic British units are still used in everyday life in the English-speaking countries. We can convert between the metric and British systems of units by using one of the following relations, which are shown in Fig. 1.

1 inch (in.) = 2.54 centimeter (cm)
1 foot (ft) = 30.5 cm
1 meter (m) = 39.37 in.

The first relation above is exact, since it is the legal definition of the inch adopted by Congress. The other two relations are sufficiently accurate for use in problems in this book.

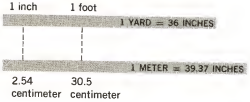


FIGURE 1

One problem which often occurs is the conversion from one unit to another of the same type but of different size. Suppose we wish to convert eight feet into its equivalent value in terms of inches. Consider the fraction 12 in./1 ft. Since this fraction has equal numerator and denominator, its value is 1. Thus, we can multiply by this fraction without changing the physical length while we do change the unit in which it is measured. In the present example we can write:

$$8 \text{ ft} = 8 \text{ ft } (12 \text{ in.}/1 \text{ ft}) = 8 \times 12 \text{ in.} = 96 \text{ in.}$$

We cancel feet in the numerator against feet in the denominator, leaving the inch as the remaining unit. The remainder of the problem consists of multiplying 8 and 12 in order to get the numerical part of the result. As a somewhat more difficult example, let us convert sixty miles (mi) per hour (hr) into feet per second (sec). We write then:

$$\begin{aligned} 60 \text{ mi/hr} &= (60 \text{ mi/hr})(5280 \text{ ft}/1 \text{ mi})(1 \text{ hr}/60 \text{ min})(1 \text{ min}/60 \text{ sec}) \\ &= \frac{60 \times 5280}{60 \times 60} \text{ ft/sec} = 88 \text{ ft/sec} \end{aligned}$$

In this case we cancel miles against miles, hours against hours, and minutes against minutes. The remaining units are ft/sec. The numerical part of the result is $(60 \times 5280)/(60 \times 60)$, which equals 88.

We will now define the unit of time. Suppose that we imagine erecting a vertical plane oriented north and south. The instant that the sun crosses this plane is called local noon. The time interval between two successive local noons is called a *solar day*. Since the length of a solar day varies slightly throughout the

year, the standard of time is defined as the average solar day or the *mean solar day*. This is too long a period of time for most measurements, so we subdivide the mean solar day into 24 hours. Each hour is further subdivided into 60 minutes and a minute is subdivided into 60 seconds. Thus, in a mean solar day there are $24 \times 60 \times 60 = 86{,}400$ seconds. The unit of time universally used in scientific work is the *second*, which is defined as 1/86,400 of a mean solar day. Fortunately, the second is also used in the English-speaking countries, so that there is no problem of conversion in this case.

2.2 Speed and acceleration in straight-line motion

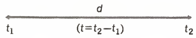
We all have an idea of what is meant by the word "speed." For instance, we say that we are driving at 40 miles per hour or that our average speed during a trip was 35 miles per hour. Similarly, when we speed up a car we have a feeling of acceleration, which we associate with the increase of speed. In this section we will make these ideas quantitative, although we will limit ourselves to the case of motion in a straight line. In Sec. 2.4 we will consider motions in a curved path.

Suppose that we travel a distance d in a straight line during a time t as shown in Fig. 2. Then we *define* our average speed $\bar{v}$ during this time interval by the equation:

$$\bar{v} = \frac{d}{t} \qquad (1)$$

It should be noted here that it is customary to indicate an average value of a quantity by putting a line over its symbol. This is the reason that the average speed was written as $\bar{v}$.

FIGURE 2



Clearly the units of average speed can be any unit of distance divided by a unit of time. In scientific work the most common unit of speed is the meter per second (m/sec), although units such as the cm/sec, ft/sec, and mi/hr (mph) are in common use. We can use algebra to re-write Eq. 1 so as to solve either for the distance covered at a certain constant speed during a given time, or we can find the time required to cover a specified distance at a given constant speed. We then obtain the following two equations:

$$d = \bar{v}t \tag{2}$$

$$t = \frac{d}{\bar{v}} \tag{3}$$

The reader should note, however, that Eq. 2 and 3 are simply different ways of writing Eq. 1.

As an example, suppose that we run 200 meters in 20 seconds. Our average speed is then given by $\bar{v} = 200$ m/20 sec $= 10$ m/sec. Suppose now that we would like to compute the distance that we would run at this same average speed during 4 minutes. Since 4 minutes is the same as 240 seconds, from Eq. 2 we see that the distance would be $d = 10 \times 240$ meters, which is a great deal more than a mile. Similarly, the time to cover 1500 meters (the metric mile) would be given by Eq. 3 by $t = 1500/10 = 150$ sec $= 2{:}30$ min, which is more than a minute below the world record for this foot race.

In many cases our speed will vary from instant to instant. For example, in driving a car our speed is zero when we are stopped by a traffic light and will have other values as we start up. We might ask what meaning we can give to the word speed in this case. In Eq. 1, if we use a very short distance d and a correspondingly small time t, we will find experimentally that the ratio of d to t will be about constant. Depending on the accuracy we require, the average speed d/t over a short distance will be sufficiently close to the instantaneous speed during any part of the time interval t for us to say that they are the same. For greater accuracy we may have to use a shorter distance near the point which concerns us. Thus, we *define* the *instantaneous*

FIGURE 3

speed in the vicinity of a point as the average speed over a small distance near that point. The speedometer on your car indicates the average speed of your car over a short distance, which is almost exactly your instantaneous speed.

Suppose that your speed changes from a value v_1 to a value v_2 during a time t as shown in Fig. 3. We say then that you have been accelerated during this time interval and *define* your *average acceleration* $\bar{a}$ during this time interval by the equation:

$$\bar{a} = \frac{v_2 - v_1}{t} \tag{4}$$

As we discussed above, if your speed is varying rapidly, we must use a small time interval to calculate your actual or *instantaneous acceleration* near a point. All examples in this book will deal with cases of constant acceleration, so that average and instantaneous accelerations will be the same. From now on we will ignore the distinction.

From Eq. 4 we see that any unit of speed divided by a unit of time will make a unit of acceleration. In this book we will measure speed in m/sec and time in seconds, so that the unit of acceleration is m/sec per second. This is usually abbreviated m/sec/sec or m/sec^2.

Suppose that your speed increased from 30 m/sec to 100 m/sec during 5 sec. Then your acceleration would be:

$$\bar{a} = \frac{100 - 30}{5} = 14 \text{ m/sec}^2$$

If your speed dropped from 40 m/sec to 10 m/sec during 6 seconds, your acceleration would then be:

$$\bar{a} = \frac{10 - 40}{6} = -5 \text{ m/sec}^2$$

In this case the acceleration is negative, which is usually called a *deceleration*. Generally, however, the word acceleration means either an increase or decrease in speed.

2.3 Kinematics of straight-line motion

The word *kinematics* means the description of motion without regard for its origin. In this section we will only deal with situations in which the acceleration is constant. If we multiply both sides of Eq. 4 by t we obtain:

$$at = v_2 - v_1$$

From this we see that v_2 is given by the equation:

$$v_2 = v_1 + at \tag{5}$$

Since the speed changes uniformly with time, we see from Fig. 4 that the average speed during the time interval t is simply the average of initial and final speeds. Thus we can write for the average speed during the time t

$$\bar{v} = \frac{v_1 + v_2}{2} \tag{6}$$

From Eq. 1, the distance covered during the time t is given by $d = \bar{v}t$. If we use Eqs. 1, 5, and 6 we can solve any problem in-

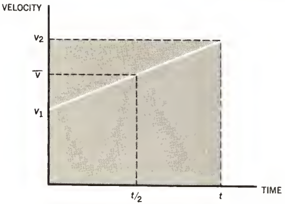


FIGURE 4

volving motion with constant acceleration. Depending on the given information, we must begin with one of these three equations and begin solving in turn for the unknown quantities.

Suppose an object with an initial speed of 10 m/sec is subjected to an acceleration of 3 m/sec^2 for a time of 5 sec. The problem is to find its speed at the end of the time interval and the distance it covers during the time interval. In this case we can write that $v_1 = 10$, $a = 3$, and $t = 5$. Using Eq. 5 we find for the final velocity:

$$v_2 = 10 + 3 \times 5 = 25 \text{ m/sec}$$

Although the average velocity was not asked for, we must use Eq. 6 to find it, obtaining:

$$\bar{v} = \frac{10 + 25}{2} = 17.5 \text{ m/sec}$$

Now from Eq. 1 we find for the distance covered:

$$d = 17.5 \times 5 = 87.5 \text{ m}$$

Two features of this example should be noted. It is usually worthwhile to identify the given information with the standard letters used in the equations, such as v_1, a, and t. Also, in many cases one must compute quantities which are not asked for, as was the case here when we computed the value of the average velocity.

Because it is so common in everyday life, an important special case of motion in a straight line with constant acceleration is motion under the influence of gravity. (It should be pointed out that the acceleration of gravity can only be considered constant if the distances involved are very small compared to the radius of the earth, which is 4000 miles.) Experimentally it is found that the acceleration of gravity, usually denoted by g, is approximately 9.8 m/sec^2 or 32 ft/sec^2, although there is a small variation from one point to another on the surface of the earth. The reason for this variation will become apparent in Sec. 2.9, where we will discuss Newton's law of gravitation.

Consider the following problem, which is amusing but not really typical. A stone is released from rest near the top of a

Skyscraper

FIGURE 5

building and requires 8 seconds to fall to the pavement below as shown in Fig. 5. What is the address of the building? This is an extreme example of a problem in which there is no apparent relation between the facts given and the result required. In this case we must use the facts given to calculate whatever quantities we can, in the hope that eventually the way to the answer will become clear. This is a case of motion with constant acceleration (due to gravity) so that $a = 32$ ft/sec^2. Also $v_1 = 0$. From Eq. 5 we have:

$$v_2 = 32t = 256 \text{ ft/sec}$$

From Eq. 6 we find:

$$\bar{v} = \frac{v_2}{2} = 128 \text{ ft/sec}$$

From Eq. 1 we have:

$$d = \bar{v}t = 128 \times 8 = 1024 \text{ ft}$$

Thus we find that the height of the building from the point at which the stone was released is 1024 ft. Only one building in the world satisfies this condition, namely, the Empire State Building located at 34th St. and Fifth Ave., New York City, which is the answer to our problem. This was another example of a situation where we had to compute the values of quantities which were not called for in the problem so that finally we got the required information.

*2.4 Velocity and acceleration using vectors

Quite evidently not all motions take place in a straight line, which was the case discussed above. In this section we will consider motion in curved paths. This will involve more general definitions of kinematic quantities as well as a study of vectors.

Consider walking along two sides of a rectangle along the path ABC as compared to walking along the diagonal AC, as

FIGURE 6

shown in Fig. 6. Evidently the overall change in our position is the same in either case. When we wish to describe distance moved in a certain direction, we use the word *displacement.* We lay off graphically each of the displacements AB, BC, and AC as arrows by choosing a scale such that the length of the arrow represents the distance covered, and each arrow's direction indicated the direction in which the distance was covered. Quantities such as these which require a specification of both a magnitude and a direction are called *vectors.* Examples of vectors which we will meet later in this book are velocity, acceleration, and force. Quantities such as time and distance, which do not have directional properties, are called *scalars.* Thus, distance is the scalar magnitude of the displacement vector. Algebra, for instance, deals with scalars, as does most of mathematics. In the following paragraph we will develop the special rule by which vectors are added, which is quite different from the addition of ordinary numbers or scalars.

If we refer to Fig. 6, let us choose a scale such that 1 meter equals 1 inch. The three displacements shown in Fig. 7 are

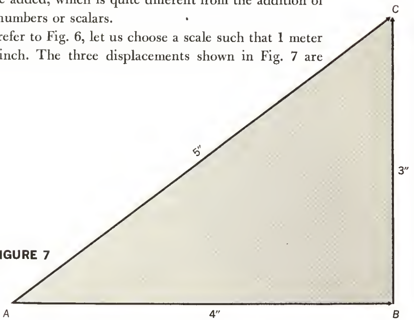


FIGURE 7

drawn to this scale. Since the effect of the two displacements *AB* and *BC* in succession is the same as the single displacement *AC*, we *define AC* to be the *sum* of *AB* and *BC*. In order to find the sum of *AB* and *BC*, we first lay off *AB* to scale in its proper direction. Then from the arrowhead of *AB* we lay off *BC* to scale. The arrow drawn from the beginning point of the arrow *AB* to the head of the arrow *BC* is then defined as the vector sum of *AB* and *BC*. The magnitude and direction of the sum *AC* can then be obtained by direct measurement and is found to be 5 inches. However, since the three vectors form a triangle, the methods of trigonometry can be used if the reader is familiar with them. In this case we have a right triangle, so that the Pythagorean formula can be used. In this book, however, graphical methods will be accurate enough.

Exactly the same process is used when more than two vectors are involved or when the vectors are not at right angles. We first choose a suitable scale, which will keep the diagram on the paper. First we lay off the first vector to scale in its proper direction. Then we lay off the second vector to scale in its own direction, beginning at the arrowhead of the first vector. The third vector is then laid off from the arrowhead of the second vector, and so forth. The vector drawn from the beginning point of the first vector to the arrowhead of the last vector is then the sum of all of these vectors. It should be noted that the order in which we decide to lay off the vectors has no effect of the result. An example is shown in Fig. 8.

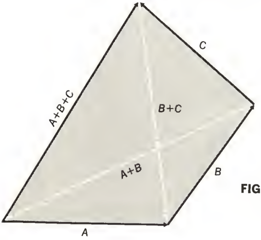


FIGURE 8

We must now consider what we will mean by the negative of a vector, so that we can perform vector subtraction. Since motion to the north is just opposite to motion to the south, it seems reasonable to *define* the negative of a given vector as an arrow with the same length and direction, but with the arrowhead at the opposite end. Suppose **A** and **B** are two vectors and we wish to calculate **A** − **B**. (Following common usage, vectors in this book will be symbolized by bold-face type.) This can be written as **A** + (−**B**). In Fig. 9 we show **A**, **B**, −**B**, and **A** + (−**B**). Thus the subtraction of vectors is virtually the same as the addition of vectors.

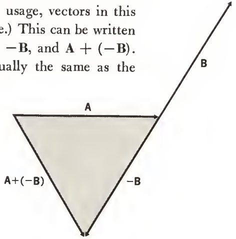


FIGURE 9

As an example of vector addition, suppose that a ship sails 40 miles east and then sails 50 miles northeast, and we are expected to find the resultant displacement. Let us choose a scale such that 10 miles equals 1 centimeter. The vector diagram is then shown in Fig. 10. By direct measurement we find that the resultant displacement is 85 miles in a direction 26° north of east.

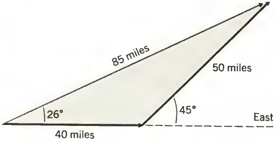


FIGURE 10

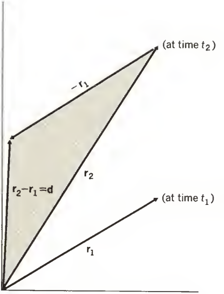


FIGURE 11

As an example of vector subtraction, suppose that the location of a particle relative to a point taken as the origin is specified by two vectors, $\mathbf{r}_1$ and $\mathbf{r}_2$, drawn from the origin at two instants t_1 and t_2 as shown in Fig. 11. The displacement during this time interval is then $\mathbf{r}_2 - \mathbf{r}_1$ and is shown as the vector $\mathbf{d}$ in the diagram. We then *define* the *average velocity* $\bar{\mathbf{v}}$ of the point during this time interval by the equation:

$$\bar{\mathbf{v}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{t_2 - t_1} \tag{7}$$

Note that velocity is a vector, with properties of magnitude and direction—in contrast to speed, which has only the property of magnitude. Although the two words are used interchangeably in everyday life, speed and velocity have distinctly different meanings in physics. We will run into examples of this sort again, in which words which are ordinarily considered to mean the same thing will be given different technical meanings. If the velocity is not constant in magnitude and direction, we follow the same procedure in relating instantaneous velocity to average velocity as was done in Sec. 2.2 in the case of instantaneous and average speed. We take a sufficiently small time interval so that the velocity does not change appreciably and then use Eq. 7 to

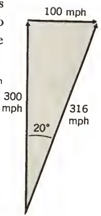
calculate the average velocity during this time interval. The average velocity during this small time interval is then defined to be the instantaneous velocity throughout the interval. Obviously, if greater accuracy is desired, a smaller time interval must be chosen.

Since velocities are vectors, they are added in the same way as displacements. As an example, suppose that an airplane maintains a speed of 300 miles per hour (mph) relative to the air in a northward direction, while the air itself is moving with a speed of 100 mph eastward relative to the ground. We wish to find the velocity of the airplane relative to the ground. If we choose a scale such that 1 cm equals 50 mph, the vector diagram for this situation is shown in Fig. 12. By direct measurement we find the airplane has a velocity of 316 mph relative to the ground in a direction 20° east of north.

Earlier we defined acceleration as the change in speed per unit of time for the case of motion in a straight line. Since velocity has properties of both magnitude and direction, it is reasonable to generalize and state that a change in either of these properties represents an acceleration. Thus, we *define* the *average acceleration* $\bar{\mathbf{a}}$ when the velocity changes from $\mathbf{v}_1$ to $\mathbf{v}_2$ during a time interval t by the equation:

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t} \tag{8}$$

As we discussed earlier in this section, if the acceleration is variable, we must choose a very small time interval if we wish to calculate the *instantaneous acceleration.* Similarly, the greater the accuracy we require, the smaller the time interval must be.



SCALE: 50 mph = 1 cm

FIGURE 12

In Sec. 2.2 we treated the special case in which the only property of the velocity which changed was its magnitude, since the motion was in a straight line. Clearly Eq. 8 reduces to Eq. 4. In Sec. 2.10 we will treat the other special case, in which the magnitude of the velocity remains constant while its direction continually changes. The general case in which both properties of the velocity change simultaneously is taken up in more advanced books on mechanics.

2.5 Newton's laws of motion

So far in this chapter we have studied the motions of particles without regard for the causes of these motions or the influences which might change the motions. In this section we will discuss forces and their effects on motions, as well as the concepts of mass and weight. The three laws of Newton are the heart of of dynamics, so that the student should be especially attentive in studying this section.

We will now undertake the delicate task of defining mass in a way first stated by Mach late in the nineteenth century. We all have some physical feeling for inertia, which is a qualitative term for mass. Even though they are about the same in size, the impact of a medicine ball is greater than the impact of a basketball, since the medicine ball has more inertia. We will now give a quantitative meaning to the concept of mass.

A pair of particles may interact in a variety of ways. They may be connected by a rod, so that we would say that they interact mechanically. Similarly, they might be connected by an elastic spring and thus interact elastically. If the particles are electrified or magnetized, other interactions could take place. Repeating something from the first paragraph of this section, the effect of an interaction is to change the motions of the particles.

Let us consider various interactions between a given pair of particles, which we will label 1 and 2. As a generalization based upon many experiments, we find that the accelerations of the

FIGURE 13

FIGURE 14

two particles are always along the line joining the particles and oppositely directed. A typical situation is shown in Fig. 13. In this case the particles are pushing one another apart. (They might be connected by a spring which is expanding.) The accelerations of the two particles might be toward one another, as is shown in Fig. 14. (the particles might be connected by a spring which is contracting.) In either case the accelerations of the two particles are along the same straight line but in opposite directions. This is found to be true experimentally for all possible interactions between a given pair of particles.

In addition to the directional properties of the accelerations when two particles interact, we find also that the magnitudes of the accelerations of the two particles are always proportional. This relation is found to be true regardless of the type of interaction or the values of other variables, such as the particles' separation, velocities, temperatures, and so forth.

Suppose we let $\mathbf{a}_{12}$ stand for the acceleration of particle 1 caused by the action of particle 2, and let $\mathbf{a}_{21}$ stand for the acceleration of particle 2 caused by the action of particle 1. Experimentally we find that $\mathbf{a}_{12}$ is proportional to $\mathbf{a}_{21}$. If we introduce a constant of proportionality k, we can then write that:

$$\mathbf{a}_{12} = -k\mathbf{a}_{21} \tag{9}$$

The constant k in Eq. 9 is found to describe *all* interactions between this pair of particles, regardless of how we may change the conditions of the experiment. This joint and universal property of the bodies is then called their *mass-ratio*. If particle

1 is taken to be unit mass by definition, then k is said to be the *mass*, m, of particle 2, since a ratio with unity in the denominator equals its numerator. In principle, then, masses are compared by letting them interact in any manner and using Eq. 9 to calculate their mass-ratio. As is common in science, less direct methods are more accurate and easier, as will be described below.

The *standard of mass* throughout the world is the kilogram (kg), which was originally defined by the French in 1789 as the mass of a cube of water 10 cm to an edge. Thus, the volume of the cube is 1000 cubic centimeters. Fortunately, however, they constructed a piece of platinum-iridium alloy which was supposed to agree with this definition, and this has been the fundamental definition of the kilogram ever since. Common submultiples of the kilogram are the gram and the milligram. In the English-speaking countries the common unit of mass is the pound (lb), which is defined legally by the relation:

$$1 \text{ kg} = 2.20 \text{ lb}$$

In order to save confusion, in this book only the kilogram will be used as a unit of mass. If situations occur in which other units of mass appear, the reader is urged first to convert all masses to kilograms before undertaking any calculations.

Now let a given influence act on two different masses which have been measured as described in the preceding paragraph. For instance, the influence might be the pull of a spring stretched to a known length. (Note that we are assuming that only a single influence acts on the bodies in question.) We find by experiment that the body with greater mass acquires a smaller acceleration. More exactly, for a given influence acting on several particles one by one, we find that the product of the mass of the particle and the acceleration it receives is a constant. Clearly there is an important relation between the acting influence and the product of the mass and the acceleration of the body on which the influence acts.

In place of the word influence we will now use the word *force* and symbolize it by the letter F. We will *define* the force

acting on a body of mass m and receiving an acceleration a as being equal to the product ma. This is known as *Newton's second law of motion.* In the form of an equation we can write:

$$F = ma \tag{10}$$

We use Eq. 10 to define units of force. In words, unit force is that force which when applied to a body of unit mass gives the body unit acceleration. In the metric system, mass is measured in kilograms and acceleration in m/sec^2. The unit of force in this system is called the *newton* (nt) and is that force which will give a body with a mass of 1 kg an acceleration of 1 m/sec^2. This is the only unit of force which will be used in this book.

Since we have discussed a case in which only a single force acts on a body, this force is the net or resultant force on the body. In more complicated situations several forces may act. Then the letter F in Eq. 10 stands for the net or resultant force acting *on* the body which has the mass m and acquires the acceleration a. It should further be pointed out that since $\mathbf{a}$ is a vector, $\mathbf{F}$ is also a vector parallel to $\mathbf{a}$. More generally then, Eq. 10 should be written in the form:

$$\mathbf{F} = m\mathbf{a} \tag{11}$$

Forces are added just like any other vectors to obtain the net or resultant force on the mass m. The graphical methods explained in Sec. 2.4 are easy and are sufficiently accurate for any problems in this book. For the special case where the forces act along the same line, they are added or subtracted just like scalars in order to find their resultant. Examples below and problems at the end of this chapter should help the reader gain familiarity with the application of Eq. 10 or 11.

At this point we will consider the special case of the net force on a body being zero. From Eq. 10 or 11 we then find that the acceleration is zero. If we are interested in straight line motion, we see that zero acceleration means the same as constant (including zero) speed. In the more general case zero acceleration means no change in the magnitude or direction of the ve-

locity of the particle. We can now state *Newton's first law of motion* in the form:

If no net force acts on a body, it will remain at rest if it is originally at rest, or it will continue moving in a straight line with constant speed.

In this statement of Newton's first law, the key word is the adjective "net" before the word force. It is perfectly possible for two or more forces to act on a body in such way that they cancel one another. In this case, no net force acts on the body and it obeys the first law. In place of the word "net" we could just as well have used "resultant" or "unbalanced," since they have the same meaning in this context. To sum up, the first law is a special case of the second law of motion, since it tells us what happens when zero net force acts on a body.

We will now take up several examples of problems based on the second law. Suppose we wish to find the force required to give a body with a mass of 2 kg an acceleration of 5 m/sec^2. From Eq. 10 we find that $F = 2 \times 5 = 10$ nt. On the other hand we might wish to know the acceleration acquired by a body of mass 4 kg when it is subjected to a force of 15 nt. We have then from Eq. 10 that a equals $\frac{15}{4} = 3.75$ m/sec^2. In another case we might have a force of 20 nt acting to the right and a force of 8 nt acting to the left on a body of mass 4 kg. The net force on the body is then $(20 - 8) = 12$ nt. The acceleration of the body is then $\frac{12}{4} = 3$ m/sec^2.

In everyday life the words weight and mass are used interchangeably, but this is not the case in physics. The word "mass" is used to describe the inertial properties of bodies and is defined quantitatively earlier in this section. Mass is a universal property of a body, the same anywhere in the universe and the same for any type of interaction in which it might take part. (Here we neglect the changes predicted by the theory of relativity, which have no practical effect on bodies of ordinary size.) "Weight," on the other hand, is used to describe the effect of gravity on the mass of a body and is thus a force. We define

the *weight* of a body as the force of gravity on the body. Thus, weight is measured in newtons, while mass is measured in kilograms. If we let g stand for the acceleration of gravity at some point on the earth and W stand for the weight of a body of mass m placed at that point, Eq. 10 takes the form:

$$W = mg \tag{12}$$

Since $g = 9.8 \text{ m/sec}^2$ approximately, a body of mass 2 kg has a weight of about $2 \times 9.8 = 19.6$ nt.

At a given location it is found experimentally that the acceleration of gravity g is the same for all bodies. We will consider two bodies, labelled 1 and 2, located at the same place. From Eq. 12 the weight of body 1 is given by:

$$W_1 = m_1 g \tag{13}$$

Similarly the weight of body 2 is:

$$W_2 = m_2 g \tag{14}$$

It should be noted that in Eq. 13–14 we used the fact that the acceleration of gravity g is the same for both bodies, since they are at the same point on the earth. If we now divide Eq. 13 by Eq. 14, we find:

$$\frac{W_1}{W_2} = \frac{m_1}{m_2} \tag{15}$$

A balance or scale actually compares weights (forces), but from Eq. 15 we see that the ratio of weights of two objects is the same as the ratio of their masses.

In practice, we use a set of calibrated masses on a balance in determining the mass of an unknown body. While this method is not the same as the direct comparison of masses in an inertial experiment, as we discussed earlier in this section, a balance is commonly used because of its ease and accuracy. Here we might note that the English language has the two nouns, weight and mass, but only a single verb, to weigh. In laboratory work, when we weigh an object and state that it weighs 2 kg, we really mean that it has a mass of 2 kg and therefore a weight of 19.6 nt.

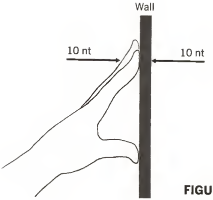


FIGURE 15

Assuming that we know the local acceleration of gravity, if we state the value of either the mass or weight for a body, we also imply a knowledge of the other quantity. This is an example of a case in which two words are used differently in physics even though they are considered the same in everyday life.

Newton's *third law of motion* is quite separate from the first two laws, since it deals with forces acting between a pair of bodies, while the first two laws deal *only* with the forces acting *on* a given body. We can state the third law in the form:

When body A exerts a force on body B, body B exerts an exactly equal but oppositely directed force on body A.

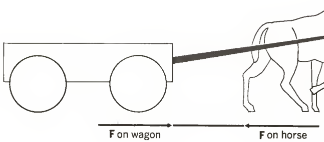
In different words, forces between bodies always occur in equal and opposite pairs. If I push against the wall with a force of 10 nt as shown in Fig. 15 the wall pushes back on me with a force

FIGURE 16

of 10 nt. This law holds whether either of the bodies is at rest, moving, or undergoing an acceleration, and is quite independent of the other two laws.

It may sound odd to say that I can walk across the floor only because the floor pushes on my shoes, but that is the case. As shown in Fig. 16, if I push to the right on the floor, the third law tells me that the floor will push to the left on me. It is this force to the left which allows me to move, according to the second law and Eq. 10. If I tried to walk across slick ice with very little friction, I could not exert a force to the right on the ice, so that the ice would not exert a force to the left on me in return, and thus I could not walk across the ice.

FIGURE 17



As another example of the third law, let us consider the following paradoxical question, which is based on the diagram of a horse and wagon shown in Fig. 17. "Is it true that the wagon pulls back on the horse with exactly the same force at all times that the horse pulls forward on the wagon? If so, how is motion produced? If not, how do you reconcile your answer with the third law?" The third law is obeyed in all situations, so it is true that the wagon always pulls back on the horse with the same force that the horse pulls forward on the wagon. Motion of the wagon, including acceleration, is produced by the forces acting *on* the wagon, which would include the force exerted forward on the wagon by the horse and the retarding force exerted on the wagon's wheels by the ground. Thus, the second and third laws are applied independently in analyzing situations of this sort.

*2.6 Equilibrium of a particle

When the net force on a particle is zero, we say that the particle is in *equilibrium.* Since in this case the acceleration of the particle is zero, it either remains at rest or continues moving in a straight line with constant speed. While most examples of equilibrium are static cases, for which the speed is zero, there are important examples in which the speed is constant. For instance, when a car travels down a straight road at constant speed, the force exerted forward on the tires is just equal and opposite to the retarding forces exerted by air resistance and friction. Similarly, after a parachutist has fallen a reasonably short distance, he reaches a constant speed. In this case the downward force of gravity and the upward retarding force of the parachute just balance one another.

In the examples discussed in the preceding paragraph the forces involved were along the same straight line, so that they could be added and subtracted just like ordinary numbers. If the forces are not along the same line, we must use the methods of vector addition treated in Sec. 2.4. If a given set of forces is to produce equilibrium, their vector sum must be zero. Thus, if we use the graphical method of adding vectors explained in Sec. 2.4, the head of the last arrow must end on the beginning point of the first vector. A typical diagram of this sort is shown in Fig. 18. Similarly, if we are required to find the magnitude and

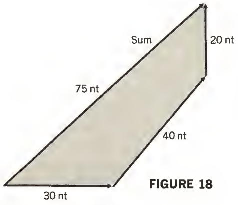


FIGURE 18

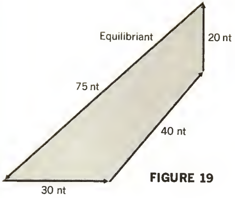


FIGURE 19

direction of an unknown force—called the *equilibriant*—which will produce equilibrium when combined with a given set of forces, we lay off the given vectors graphically. Then the force which completes the chain of forces so as to give zero net force is the required answer. An example of this sort is shown in Fig. 19. Thus, the 75 nt equilibriant of Fig. 19 just balances the 75 nt resultant of Fig. 18.

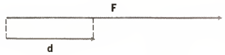
2.7 Work, energy, and power

Suppose that a constant force **F** acts through a distance **d** parallel to the force, as shown in Fig. 20. (The general case in which the force and the distance through which it moves are not parallel or are variable will not be treated in this book.) We *define* the *work* W done by the force by the equation:

$$W = Fd \tag{16}$$

where F and d are the scalar values of **F** and **d**. The reader should note that this is the technical meaning of the word "work," even

FIGURE 20



though other meanings are used in everyday life. In the metric system which we are using in this book the force is measured in newtons and the distance in meters. Thus, the unit of work is 1 nt-m. Since work is so important, this unit is called the *joule* after a famous physicist whose work is described in the following chapter. We have then:

$$1 \text{ joule} = 1 \text{ nt-m}$$

The rate at which work is done by a force is also of great interest in science. Again we will restrict our discussion to a constant force. If the force does an amount of work W during a time t, we define the *power* P in this situation by the equation:

$$P = \frac{W}{t} \tag{17}$$

Again it should be pointed out that this is the specific technical meaning of the word "power," although there are other common meanings. In the metric system we measure work in joules and time in seconds, so the unit of power is the joule/sec. Since this unit occurs frequently in physics, it is named the *watt* after the builder of one of the first steam-engines. We can write then:

$$1 \text{ watt} = 1 \text{ joule/sec}$$

When work is done by a force on a body, one or more properties of the body will change, such as its speed, its height, its temperature and so forth. As a general definition we state that each change in a property of the body is accompanied by a change in a corresponding form of energy. Thus, when work is done on a body one or more forms of its energy will increase. It is usually most convenient in analyzing various forms of energy to arrange the way in which force is applied so that only a single property of the body is changed. In this case all of the work done on the body goes to increase one particular form of the body's energy.

Consider raising a body a vertical distance h, starting with the body at rest and ending with the body at rest as shown in Fig. 21. Thus, the only property of the body which has changed is its vertical position. Since the work to accomplish this change

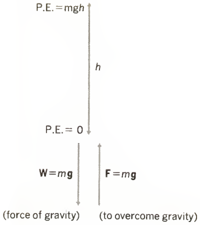


FIGURE 21

has been done against the force of gravity, we say that we have increased the *gravitational potential energy* of the body. From Eq. 13 of Sec. 2.5 we know that the weight of the body whose mass is m is mg. Thus, the force required to raise the body a distance h against the force of gravity is numerically equal to mg, and the work done by this force is then $(mg)h$. From the general definition of energy in the preceding paragraph, we find for the increase in the gravitational potential energy of the body (abbreviated P.E.) the value:

$$\text{increase in P.E.} = mgh \tag{18}$$

The work which has been done to raise the energy of the body is not lost, but rather is stored and is available to do work. In this particular situation we might think of the pile-driver. Work is done to raise a heavy weight, and the work is recovered when the weight drops and drives the piling into the ground.

Let us now consider a constant force F accelerating a body of mass m from rest to some final speed v during a time t, as shown in Fig. 22. In this case the definition of acceleration given

FIGURE 22

by Eq. 4 reduces to $a = v/t$, since $v_1 = 0$ and we have let $v_2 = v$. Thus, Eq. 11 takes the form:

$$F = ma = m\frac{v}{t} \tag{19}$$

Also, the average speed in this case is given by $\bar{v} = v/2$, so that the distance covered is given by $d = (v/2)t$. The work done by the force is then given by

$$W = Fd = \frac{mv}{t} \times \frac{vt}{2} = \frac{1}{2}mv^2 \tag{20}$$

Since all of the work in this example went to increase the speed of the body, without any other property of the body changing, we say that the increase in the *kinetic energy* of the body (abbreviated K.E.) is $\frac{1}{2}mv^2$. While the discussion here of kinetic energy has dealt only with a constant force, it is found that in the general case the kinetic energy of a body is still given by $\frac{1}{2}mv^2$, regardless of the way in which the speed v was obtained. Again, the work done to accelerate a body to a speed v is not lost, but can be recovered. For instance, when you swing a hammer, the head of the hammer has kinetic energy, which is then recovered when the head hits the nail and drives it into the wood.

In some situations there can be a conversion between potential and kinetic energies. For instance, a ball might be thrown vertically upward with a speed v and a kinetic energy $\frac{1}{2}mv^2$. This energy is gradually used up against the force of gravity, so that at the peak of its path the ball has no kinetic energy, but has an equal amount of potential energy. We can write then:

$$\tfrac{1}{2}mv^2 = mgh \tag{21}$$

From Eq. 21 we can then easily calculate the maximum height reached by the ball. As the ball falls downward again it gives up potential energy and gains kinetic energy. When the ball has returned to its starting point it has lost all of its potential energy and has exactly the same kinetic energy as it had at the beginning, if we neglect losses due to air resistance. This is an example of *conservation of mechanical energy*. (We say that a quantity is

Height (meters)	Potential energy (joules)	Kinetic energy (joules)
5	98	0
4	78.4	19.6
3	58.8	39.2
2	39.2	58.8
1	19.6	78.4
0	0	98

FIGURE 23

conserved if its value remains constant.) In the example above we found that the sum of the ball's kinetic and potential energies remained constant, if we could neglect losses due to air resistance.

Suppose that a particle of mass 2 kg is released from rest and falls a distance of 5 meters. The change in the potential energy of the particle is then given by:

$$\text{decrease in P.E.} = mgh = (2 \times 9)(8 \times 5) = 98 \text{ joules}$$

This decrease in potential energy is all converted into an increase in kinetic energy, so that we have:

$$\text{increase in K.E.} = \text{decrease in P.E.} = 98 \text{ joules} = \tfrac{1}{2}mv^2$$

Since the mass of the particle is 2 kg, we easily find that the speed gained by the particle is 9.9 m/sec. Here we have neglected any effects due to air resistance, and various values of the energies of the particle are shown in Fig. 23. Neglecting air resistance is a very good approximation if the particle is made of a dense material such as a metal, but would not be if the object were a rubber balloon.

2.8 Momentum

If we use the definition of acceleration given by Eq. 4, Newton's second law of motion as expressed in Eq. 10 takes the form:

$$F = ma = m\frac{v_2 - v_1}{t} = \frac{mv_2 - mv_1}{t} \tag{22}$$

If we multiply both sides of Eq. 22 by t we find:

$$Ft = mv_2 - mv_1 \tag{23}$$

In the preceding section we found that the product of a force and the distance through which it acts is an important quantity called work. In Eq. 23 we have the product of a force and the time during which it acts, which is called the *impulse* of the force. On the right side of Eq. 23 we have the product of the mass of the particle on which the force acts and its speed at two different instants. This product is called the *momentum* p of the particle. In the form of an equation the momentum of particle of mass m and speed v is given by:

$$p = mv \tag{24}$$

Since in the general case we must replace the speed by the velocity, which is a vector, in general, momentum is a vector defined by $\mathbf{p} = m\mathbf{v}$. As Eq. 23 was derived directly from Eq. 10, it is equivalent to it. Thus, an alternative statement of Newton's second law of motion is that a given impulse of a force produces an equal increase in the momentum of the particle on which the impulse acts.

If two particles interact, the forces they exert on one another are equal but oppositely directed, according to Newton's third law of motion. If no external forces act on the particles, since the forces they exert on one another cancel each other, Eq. 23 shows us that the net impulse is zero. Thus, the momentum of the particles taken together cannot change. If one of the particles gains momentum, perhaps when the particles collide, the other particle must lose an exactly equal amount of momentum. Thus, in interactions between pairs of particles their total momentum remains constant, which is known as the principle of *conservation of linear momentum.*

As an example of the conservation of linear momentum when two particles collide, consider the situation shown in Fig. 24. Here a putty ball of mass 0.5 kg and speed 10 m/sec strikes a

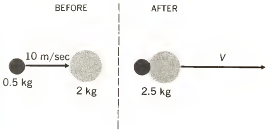


FIGURE 24

motionless object of mass 2 kg. After the impact the two objects move off together with the same speed v, which we would like to calculate. Assuming that we can neglect all external forces, momentum is conserved in this collision, so that its value is the same before and after the collision. We can write then:

$$\underset{\textit{before}}{0.5 \times 10 + 0} = \underset{\textit{after}}{(0.5 + 2)v} = 2.5v$$

When we solve the equation above for v, we find that $v = 2\text{m/sec}$.

A more interesting application of the principle of conservation of momentum is provided by a rocket or a jet engine. The burning fuel expands rapidly and is forced out of the rear of the rocket as shown in Fig. 25. Thus, rearward momentum is gained by the burned fuel. If we treat the rocket and its fuel as an isolated system and neglect air resistance, the body of the rocket must gain exactly the same amount of momentum in the forward direction. This forces the rocket forward. Even though the particles in the burned fuel are light, they are ejected with considerable velocity, so that the rocket experiences a large thrust.

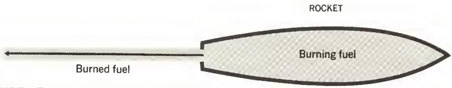


FIGURE 25

2.9 The law of gravitation

By the middle of the seventeenth century a lot of experimental information was available about the motions of the planets. Most notably, Kepler had discovered that the planets move about the sun in elliptical orbits. Using this *heliocentric* (from the Greek *helios*, meaning sun) theory of the solar system, Kepler was able to account for the observations better than any previous theory. It was Newton who solved the problem of predicting these empirical laws from the basic laws of motion.

Since an ellipse is a curved path, both the magnitude and the direction of a planet's motion is continually changing, so that the planet is accelerated. According to Newton's second law of motion a force is always required to accelerate a particle. Since it was not possible to measure directly the force between a planet and the sun, Newton had to guess the nature of the force and then see if this force would actually predict the observed motions of the planets. He concluded that the planetary motions could be explained if the force between any pair of particles of masses m_1 and m_2 separated a distance d, as shown in Fig. 26, was an attraction proportional to m_1m_2/d^2. If we introduce a constant of proportionality G, we can write the law of gravitation in the form:

$$F = G\frac{m_1m_2}{d^2} \tag{25}$$

Since gravitational forces between ordinary-sized objects are exceedingly small, Newton had no way to verify Eq. 25 directly. However, this force law did predict correctly the motions for planets and satellites—an excellent but indirect confirmation of the law.

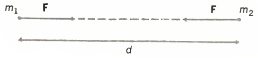


FIGURE 26

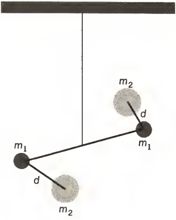


FIGURE 27

In 1798 an English scientist, Henry Cavendish, devised a delicate device to study Eq. 25 and found excellent agreement with this equation. The apparatus used by Cavendish is shown in Fig. 27. Two small masses attached to the ends of a light crossbar are attracted towards the larger masses. The forces between the masses twist the fiber supporting the crossbar through a measurable angle. By a fairly simple calculation, the force between each small mass and its neighboring large mass can be computed. In this way Cavendish verified Eq. 25 and determined the value of the constant G to within an accuracy of about one percent of the best modern value, which was a remarkable achievement in view of the crudity of his apparatus. If we use the metric system, with mass measured in kg, distance in meters, and force in newtons, the constant G is found to have the value of 6.67×10^{-11} nt-m^2/kg^2.

So far our consideration of gravitational forces has been restricted to objects which are very small compared to their distance of separation, so that the objects could be treated as mathematical points. The diameter of the sun is about one million miles, while the earth's distance from the sun is about 93 million miles. Thus, the sun is not really a point particle when we consider its force on the earth. This bothered Newton for many years, until he finally proved that two spherical and

uniform objects attract one another as if their masses were concentrated at their centers. Since both the sun and the earth are very nearly uniform bodies, we can quite accurately discuss their motions as if the mass of each were concentrated at its center. Similar remarks apply to satellites, which are discussed in the following section.

*2.10 Uniform circular motion and satellites

The same force of gravitation which Newton postulated to explain the motions of the planets also accounts for the motions of man-made satellites. The general case of elliptical orbits is beyond the scope of this book, so we will consider only the case of circular orbits. Fortunately, the orbits of most planets and satellites are very nearly circular, so that we will introduce little error by this simplification.

Suppose a particle moves in a circular path of radius r at a constant speed v. Since the direction of the motion is continually changing, the particle is shown at two instants a time t apart in Fig. 28. (Note here that $v_1 = v_2 = v$, numerically.) During the time t the particle moves a distance vt along the arc of the circle. However, if the time is taken to be sufficiently small, the length of the arc and the chord are very nearly the same. Since we have similar triangles, we can write:

$$\frac{v_2 - v_1}{v} = \frac{vt}{r} \tag{26}$$

From Eq. 26 the acceleration of the particle is given by:

$$a = \frac{v_2 - v_1}{t} = \frac{v^2}{r} \tag{27}$$

From Fig. 28 we see that this acceleration is directed towards the center of the circle, so that we refer to this as *centripetal acceleration*, where the magnitude of this acceleration is given by Eq. 27.

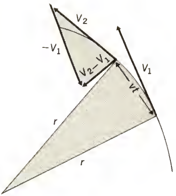


FIGURE 28

Consider now a satellite moving in a circular path of radius r (measured from the center of the earth) so that d in Eq. 25 and r in Eq. 27 correspond. Let the mass of the satellite be m_1 and the mass of the earth m_2. If we substitute Eqs. 25 and 27 into Eq. 12, we find:

$$G\frac{m_1 m_2}{r^2} = m_1 \frac{v^2}{r} \qquad \textbf{(28)}$$

After multiplying Eq. 28 by r/m_1, we obtain in place of Eq. 28:

$$\frac{Gm_2}{r} = v^2 \qquad \textbf{(29)}$$

From Eq. 29 we observe that the motion of the satellite is independent of its mass. Furthermore, the smaller the orbit of a satellite the greater its speed, which probably seems paradoxical to the reader. As a satellite is affected by the outer atmosphere and spirals towards the earth, it actually gains speed.

*2.11 Elasticity

In many cases solids, such as iron, seem to be completely rigid. We all know, however, that it is easy to bend a paper clip or a knitting needle made of iron. In this section we will be concerned with the effects of forces on the shapes and sizes of solid objects.

When forces applied at the opposite ends of a solid tend to make the body longer, we say that a *tension stress* has been applied to the body. An example might be the stretching of a guitar string when the instrument is tuned. If forces applied at the ends of a solid tend to decrease its length, a *compression stress* has been applied to the body. An example might be the compression of a truss supporting a bridge when the traffic on the bridge is heavy. In both of these cases, the forces act along the same line. If the forces do not act along the same line, the body experiences a *shear stress*, which has the effect of changing the shape of the body without changing its volume. If you push horizontally on the top of a block of Jello, you will be exerting a shear stress. Examples of the three types are shown in Fig. 29. It should be pointed out here that the actual deformations produced on real solids are usually much smaller than the ones shown in the diagram.

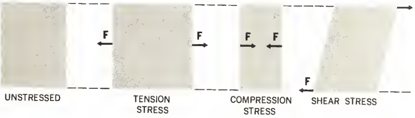


FIGURE 29

We will next consider the relation between the magnitude of a stress applied to a body and the deformation which results. This is actually a complicated problem, since it involves not only the material of the body, but its temperature, shape, and other factors. In the seventeenth century Hooke found that if the stress applied to the body was small, then the resulting deformation was proportional to the stress. This is known as *Hooke's law*, although it is actually not a law but an empirical approximation. It is commonly used, however, both because it is simple and because it is a good approximation for many materials.

Let us apply a number of forces in turn to a piece of wire, for instance, as shown in Fig. 30. The increases in length of the wire

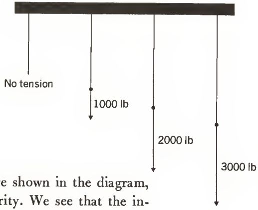


FIGURE 30

compared to its unstretched length are shown in the diagram, although they are exaggerated for clarity. We see that the increases in length (deformations) are proportional to the applied forces (stresses). If we introduce a constant of proportionality, k, we see that we can write the following equation between the applied force F and the resulting elongation x:

$$F = kx \tag{30}$$

The constant k is known as the Hooke's law constant of this particular wire. The value of k is much larger for steel than it is for rubber, for instance, and it is larger for a thick wire than for a thin wire if both are made of the same material.

Since we know that all wires break when a sufficiently large stress is applied to them, the reader might wonder how a real wire behaves as the stress applied to it is gradually increased. The graph of tension stress versus resulting elongation for a typical material is shown in Fig. 31. While the tension is small,

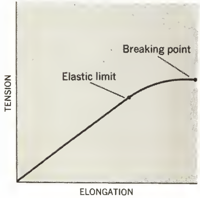


FIGURE 31

the elongation is seen to be proportional to the tension, in accord with Eq. 30. However, at a point known as the *elastic limit* the material begins to yield and Hooke's law no longer applies. In addition, if the tension is now removed, the length of the wire does not return to its original value, as would have been the case at any point on the straight portion of the graph. At an even higher value of the tension, we reach the breaking point and suddenly have two pieces of wire.

If a material obeys Hooke's law, we can easily calculate the work required to elongate the wire a given amount. As we gradually increase the force applied to the ends of the wire, the force takes on all values between $F_1 = 0$ (initially) and $F_2 = kx$ (finally). The average force applied to the wire during this process is then:

$$\bar{F} = \frac{F_1 + F_2}{2} = \frac{0 + kx}{2} = \frac{kx}{2} \tag{31}$$

The work done in producing the final elongation is then:

$$W = \bar{F}x = \frac{kx}{2}x = \frac{1}{2}kx^2 \tag{32}$$

Here it should be noted that to compress a material an amount x with a given force, the same constant k is found to be valid experimentally. Thus, both Eq. 30 and Eq. 32 are correct when x stands for a contraction rather than an elongation. In most actual cases, the elastic body is a spring and the contraction or elongation is small compared to the unstretched length of the spring, so that Hooke's law applies.

In order to demonstrate these ideas, suppose that a force of 100 newtons is required to elongate or compress a certain spring a distance of 5 centimeters. Then, the spring constant is $k = F/x = 100/0.05 = 2000$ nt/m. Also, the work required to produce this change in length of the spring is given by $W = \frac{1}{2}kx^2 = \frac{1}{2}(2000)\ (5 \times 10^{-2})^2 = 2.5$ joules. Since we know the value of k for this particular spring, we could easily compute

the force and work required to produce different changes in its length.

As we discussed in Sec. 2.7, whenever work is done on an object, we store an equal amount of energy which we can later recover. In the case of an elastic body we call this energy *elastic potential energy.* If we refer to the example in the preceding paragraph, the elastic potential energy stored is 2.5 joules. Examples of this type of energy would be the energy stored in a watch-spring or the energy stored in a sling-shot when it is pulled back. There are many other examples of the use of elastic potential energy.

Suppose that a car of mass 1000 kilograms (approximately 2200 pounds) strikes a large, immovable object, such as a boulder. If the speed of the car before impact is 32 kilometers per second (about 20 miles per hour) and the front bumper of the car has a Hooke's law constant of 10^8 newtons per meter, we might wish to calculate the compression of the bumper as the car came to rest. We would have then, by conservation of energy:

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2$$

After cancelling $\frac{1}{2}$ from each side of the equation above and solving for x, we have then:

$$x = v\sqrt{\frac{m}{k}}$$

Thus,

$$x = 32\sqrt{\frac{10^3}{10^8}} = \frac{32}{\sqrt{10^5}} = \frac{32}{316} = 0.101 \text{ meters}$$

This corresponds to a deformation of the bumper of very nearly 4 inches. It should be noted, however, that this problem was idealized, since some of the kinetic energy of the energy is transformed into heat, sound, and other forms of energy, while we assumed that all of the energy was transformed into elastic potential energy.

SUMMARY

Since all measurements involve expressing the results in terms of quantitative units, the metric system of units was discussed. Although metric units are almost universally used by scientists, the British units are still used in everyday life so the relation between the two systems of units was then explained. In the metric system multiples and sub-multiples of a given standard are related to the standard by powers of ten, which are indicated by various prefixes.

The concept of speed as the distance covered per unit of time was introduced for the simple case of motion in a straight line. Since the speed may not be constant, this led us to the related concept of acceleration, which is the change in speed per unit of time. These two concepts allowed us to describe the commonly found motion of a particle in a straight line with constant acceleration. The more general case of motion in a curved path requires a discussion of vectors which was given in a special section.

Mass was defined by considering the interactions between pairs of particles. We found that the mass of a particle governs its interactions with all other particles. The net force acting on a body was then defined as the product of the mass of the body and the acceleration it receives from the force. The weight of a body was defined as the force of gravity acting on the mass of the body. We easily justified from this the comparison of masses by comparing their weights, as is done with a balance. Newton's third law of motion, which states that forces between bodies occur in equal and oppositely-directed pairs, was then discussed. The special case of equilibrium was treated next, since it is a familiar situation.

The related concepts of work, energy, and power are used throughout all of science so they were considered in some detail. This discussion led us to the principle of conservation of mechanical energy. This principle is extended to include other forms of energy later in this book. Linear momentum and the principle of conservation of linear momentum were then treated. Newton's law of gravitation was discussed as well as the application of this law to motions of satellites about the earth. Finally, we considered elastic forces and the energy associated with them.

PROBLEMS

√ **1** *What is a milli-kilogram?* ANS.: *1 gram.*

2 *Compute the number of minutes in 1 microcentury.*

√ **3** *Convert 100 kilometers per hour into miles per hour.*
ANS.: *62.1 mph.*

4 *Compute the average speed in feet per second for a man to run the 100-yard dash in 9.0 sec.*

√ **5** *A particle is accelerated from rest to a speed of 20 m/sec during a time of 5 sec. Compute the acceleration of the particle and the distance it covers.*
ANS.: $a = 4$ *m/sec/sec*; $d = 50$ *m.*

6 *A particle falls from rest with an acceleration of 9.8 m/sec^2. After it has fallen for 5 sec, compute its speed and the distance it has fallen.*

√ **7** *A particle moving with a speed of 50 m/sec is brought to rest in a distance of 20 m. Compute the deceleration of the particle and the time before it is brought to a stop.* ANS.: *−62.5 m/sec/sec; 0.800 sec.*

8 *A boat is to be steered directly across a river which runs north and south. The speed of the boat is 10 mph relative to the water and the current in the river is 4 mph. Determine the direction in which the boat must be steered.*

√ **9** *Refer to Problem 8. If the river is 2 mi. wide, compute the time required for the boat to cross the river.* ANS.: *0.218 hr.*

10 *Compute the force required to give a mass of 3 kg an acceleration of 5 m/sec^2.*

11 *When a force of 25 nt is applied to a certain particle, it acquires an acceleration of 2 m/sec^2. What is the mass of the particle?* ANS.: *12.5 kg.*

12 *A force of 19.6 nt is applied to a particle of mass 2 kg. What is the acceleration of the particle?*

√ **13** *A rocket of mass 100 kg accelerates vertically upward with an acceleration of 5 m/sec^2. Compute the upward thrust produced by the rocket's engine.* ANS.: *1480 nt.*

14 *Compute the weight in newtons of 1 lb.*

15 *A particle is acted upon by an upward force of 20 nt and a horizontal force of 30 nt. Compute the magnitude and direction of a third force which will hold the particle in equilibrium.*

ANS.: *36 nt downward and making an angle of 56.3° with the vertical.*

16 *A force of 15 nt acts through a distance of 2 m. Compute the work done by this force.*

17 *Refer to Problem 16. If the force acts during a time of 3 sec, compute the power exerted by the force.* ANS.: *10 watts.*

18 *Compute the kinetic energy of a car of mass 1500 kg travelling at a speed of 20 m/sec.*

19 *Refer to Problem 18. Calculate the height from which the car would have to fall from rest in order to gain the same amount of energy.*

ANS.: *20.4 m.*

20 *A ball is projected vertically upward with an initial speed of 10 m/sec. Compute the maximum height to which the ball rises.*

21 *Refer to Problem 20. At the top of its path, what is the acceleration of the ball?* ANS.: *9.8 m/sec/sec downward.*

22 *Compute the speed which a car of mass 1000 kg must have in order that it will have the same momentum as a truck of mass 25,000 kg travelling at a speed of 15 m/sec.*

23 *Refer to Problem 22. Compute the kinetic energies of the car and the truck.* ANS.: *Car: 7.05 $\times$ 10^7 joules; truck: 2.82 $\times$ 10^6 joules.*

24 *Compute the force of attraction between a man of mass 80 kg and a girl of mass 50 kg. Take their distance of separation as 20 cm. (This illustrates how difficult life would be if the constant G were a billion times larger.)*

25 *Let the mass of the earth be M and its radius R. Consider the gravitational force on a particle of mass m located on the surface of the earth and show that the following relation holds:*

$$GM/R^2 = g$$

26 *Refer to Problem 25. Use the data given in this chapter to calculate the mass of the earth. (Take the radius of the earth as 6400 km.)*

27 *Calculate the speed which a satellite must have to stay in a circular orbit 1000 km from the earth's surface.* ANS.: 7.4×10^3 *m/sec.*

28 *What is a milli-micro-second?*

29 *Convert 10 pounds per cubic foot into ounces per cubic inch.* ANS.: *0.927 oz/in.*^{3}.

30 *Compute the average speed of a horse which can run a mile in 2 minutes.*

31 *If the average speed of an earthworm is 2.5 cm/min, how long would it take for the earthworm to cover a distance of 30 cm?* ANS.: *12 min.*

32 *An automobile accelerates from rest to a speed of 60 mph in 10 sec. Compute the acceleration (assumed constant) of this automobile in feet per second per second.*

Acceleration

33 *Compute the deceleration required for an automobile travelling at a speed of 45 mph to come to rest in 6 sec.* ANS.: -11 *ft/sec*^{2}.

34 *Refer to Problem 33. How far would the automobile travel while coming to rest?*

35 *A particle falls from rest with an acceleration of 32 feet per second per second. Compute its speed at the end of 4 sec.* ANS.: *128 ft/sec.*

36 *Refer to Problem 35. Compute how far the particle has fallen during the 4 sec.*

37 *A boat can travel at 6 mph relative to the water in a river in which there is a current of 2 mph. Compute how long it would take the boat to travel to a buoy 2 mi downstream and return to its starting point.* ANS.: $\frac{3}{4}$ *hr.*

38 *Compute the force necessary to give a mass of 5 kg an acceleration of 9.8 m/sec*^{2}.

39 *When a force of 30 nt is applied to a certain particle, it acquires an acceleration of 4 m/sec*^{2}. *What is the mass of the particle?* ANS.: *7.5 kg.*

40 *Refer to Problem 39. Compute the momentum of the particle after the particle has experienced the force for 2 sec, if the particle started at rest.*

41 *If a force of 20 nt is applied to a particle of mass 4 kg, what is the acceleration of the particle?* ANS.: *5 m/sec*^{2}.

42 *Refer to Problem 41. If at the start, the particle had a velocity of 3 m/sec, what is its momentum 2 sec after the force is applied?*

43 *A force of 20 nt acts through a distance of 3 m. Compute the work done by this force.* ANS.: *60 joules.*

44 *Refer to Problem 43. If the force acts during a time of 4 sec, compute the power exerted by this force.*

45 *Compute the kinetic energy of a boat of mass 10,000 kg moving at a speed of 20 km/hr.* ANS.: 1.6×10^5 *joules.*

46 *Refer to Problem 45. What force is required to bring this boat to rest in a time of 50 sec?*

47 *Refer to Problems 45 and 46. Assuming that the applied force is constant, compute the distance travelled by the boat in coming to rest.* ANS.: *142 m.*

48 *Compute the initial upward speed of a particle which reaches a maximum height of 10 m above its point of release.*

49 *Compute the kinetic energy of a car of mass 1500 kg travelling at a speed of 80 km/hr.* ANS.: 3.72×10^5 *joules.*

50 *Refer to Problem 49. If all of the kinetic energy of the car is used up in 20 sec, what is the average rate at which the car gives up energy, measured in watts?*

51 *Calculate the radius of the circular orbit which a satellite must have to travel once around the earth every 24 hours. (This sort of orbit is useful for communications satellites.)* ANS.: *26,400 miles (from center of earth).*

52 *When an object weighing 5 kg is suspended by a spring, the length of the spring changes by 2 cm. Compute the force constant of this spring.*

53 *Compute the force required to stretch a spring with a force constant of 10 nt/m a distance of 3 cm.* ANS.: *0.3 nt.*

54 *Refer to Problem 53. Compute the work necessary to produce the specified elongation, if the initial elongation is zero.*

55 *A force of 100 nt is used to compress a spring with a force constant of 5 nt/cm. Compute the amount of the compression.* ANS.: *0.2 m.*

56 *Refer to Problem 55. Compute the work required to produce the specified compression.*

57 *It is desired to store an energy of 2 joules in a spring which is to be compressed 4 cm. What must the force constant of the spring be?*

ANS.: 2.5×10^3 *nt/m.*

DISCUSSION QUESTIONS

1 *A particle is thrown vertically upward. What is its acceleration at the peak of its path? What is its acceleration when it returns to its starting point?*

2 *Do the speed and the acceleration of a particle have to be in the same direction?*

3 *What units of length are used by astronomers in expressing distances within the solar system and distances to stars?*

4 *Accelerations experienced by airplane pilots during radical maneuvers are often expressed as 5g, 8g, and so forth. What is meant by such a statement?*

5 *If a quaver is an eighth-note in music, what is a hemidemisemiquaver?*

6 *Since the acceleration of free fall is quite large, measurements on freely falling bodies are difficult to make. Discuss how Galileo was able to observe much smaller accelerations by studying balls rolling down an inclined plane.*

7 *A body is known to be accelerated. Can we say with certainty anything about the magnitude and direction of the body's velocity?*

8 *If you wished to increase the thrust of a rocket motor, would it be better to use a different fuel, more fuel, or make some other change in the motor's design?*

9 *When a man is put into orbit in a satellite, does either his weight or his mass change?*

10 *Why are larger hammer-heads used to drive larger nails and spikes?*

11 *Is it possible for two forces of magnitudes 5 and 8 units respectively to maintain a particle in equilibrium?*

12 *Is it possible for a particle to have a velocity in one direction while experiencing an acceleration in another direction?*

13 *A boy in a railroad car moving with constant speed in a straight line throws a ball vertically upward. Does the ball land in front of him, in his hand, or behind him?*

14 *The engineer of a railroad train puts on the brakes and the train comes to a stop. Why do you fall forward?*

15 *When you push hard against the wall of a room, you produce no motion, yet you do get tired. Is there something inconsistent between the definition of work used in physics and its physiological meaning? (Note: Your body does use up energy in this situation.)*

16 *The engine of your car continues to burn gasoline even when your car is moving with constant speed along a straight, horizontal highway. Why is this necessary if Newton's first law is true? Discuss any energy interchanges involved in this process.*

17 *Is it possible for a body to have energy without having any momentum? Can a body have momentum without having any energy?*

18 *Explain why a rocket can produce thrust outside the earth's atmosphere, where there is no air for the exhaust to push against.*

19 *Explain why a satellite put into orbit thousands of miles from the surface of the earth can stay in the orbit almost indefinitely, even though the satellite's fuel is all gone.*

20 *Combine Eqs. 2, 4, and 6 to derive the following equation for motion with constant acceleration:*

$$v_2^2 = v_1^2 + 2ad$$

21 *Combine Eqs. 2, 4, and 6 to derive the following equation for motion with constant acceleration:*

$$d = v_1 t + \tfrac{1}{2} at^2$$

22 *Do all substances show a simple proportionality between the applied stress and the resulting deformation, as discussed in Sec. 2.11?*

23 *Substances which flow, such as plastics, are known as rheological substances. Would you expect them to obey Hooke's law?*

24 *If a ball of putty is squeezed, would you have to do work? Could you recover this work?*

25 *When you blow up a balloon, you do work. Is it possible to recover this work?*

26 *Assume that a spring obeys Hooke's law for small compressions. Is there an obvious limit to the compression for which this law fails? What is it?*

CHAPTER THREE

THERMODYNAMICS

3.1 Temperature scales

We all have some sort of a feeling for the meaning of temperature in terms of hot and cold. However, the human senses are not very reliable in estimating temperatures. When our hands are cold, even lukewarm water feels quite hot. If a metal pan and wooden spoon are heated to the same temperature in an oven, the pan will feel hotter than the spoon because it conducts heat to the skin more rapidly. It is evident from these examples that we must use some property of nature to assign temperatures which is not as variable as our human senses.

It is a fact of ordinary experience that various properties of materials vary with temperature. When we drive a car on a hot day, the tires heat up and the air pressure in the tires increases. On a cold day it is harder to start the car's engine, both because the oil is more viscous ("thicker") and because the battery delivers energy less readily. A less obvious change is the increase in length of a metal rod with an increase in temperature, which must be allowed for in building bridges. Similarly, the volume of a liquid usually increases with temperature, so that oil tanks are not filled full to allow for this expansion. These are just a few examples of physical properties which depend on temperature.

In principle, at least, the variation of any physical property could be used to define a temperature scale, although the various scales would not agree very well. First we arbitrarily choose two temperatures which can easily be reproduced in the laboratory and assign numbers to these temperatures. In the *Celsius* or *Centigrade* scale named after a Swedish astronomer and used by scientists and most other people in the world, the temperature at which pure water freezes is defined as 0°C and the temperature at which pure water boils is defined as 100°C. In each case the measurement is to be made when the pressure of the surrounding air is 14.7 lb/in.^{2}, which is standard atmospheric pressure. On this scale the interval between the freezing and boiling points of water is subdivided into 100 degrees. The *Fahrenheit* scale is used in the English-speaking countries. Here the freezing point of water is defined as 32°F and the boiling point of water as 212°F. On the Fahrenheit scale the interval between the freezing and boiling points of water is subdivided into 180 degrees. The relation between these two temperature scales is shown in Fig. 32. From the diagram we note that each Celsius degree is $\frac{180}{100} = \frac{9}{5}$ as great as a Fahrenheit degree. Since the freezing point is defined as 32°F, we see that we must subtract 32 degrees from a Fahrenheit temperature before converting it to a Celsius temperature. We then have as the relation between the temperature F on the

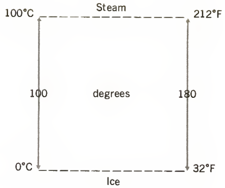


FIGURE 32

Fahrenheit scale and the corresponding temperature C on the Celsius scale the equation:

$$C = \frac{5}{9}(F - 32) \tag{33}$$

If we wish to convert a Celsius temperature to a Fahrenheit temperature, we can write Eq. 33 in the form:

$$F = \frac{9}{5}C + 32 \tag{34}$$

As an example, let us convert 68°F into the equivalent Celsius temperature. From Eq. 33, we have:

$$C = \frac{5}{9}(68 - 32) = \frac{5}{9}(36) = 20°\text{C}$$

If we wish to convert 25°C into the equivalent Fahrenheit temperature, from Eq. 34 we have:

$$F = \frac{9}{5}(25) + 32 = 45 + 32 = 77°\text{F}$$

As we will see later, negative temperatures are possible in both scales, so that Eq. 33 and 34 are valid for both positive and negative temperatures.

At this point the reader may be wondering how the variation in some physical property is used to define a temperature scale after the temperatures of the freezing and boiling points of

water have been given values. We do this by using simple proportion between the change in the property and the change in temperature. Suppose that the change in the property between 0°C and 100°C be called x, while the change in the property to reach the temperature t°C be called y. Then we *define* the unknown temperature by the equation:

$$\frac{t}{100} = \frac{y}{x} \tag{35}$$

The procedure described above amounts to assuming that the change in the physical property is proportional to the temperature change which produces it. We might therefore use the change in pressure of an enclosed gas, the change in length of a metal rod, or the change in volume of a liquid in order to devise a temperature scale. In fact, all of these properties and many others are used in practice. However, the various temperature scales derived from these and other properties do not agree perfectly except at 0°C and 100°C, since the various properties do not change in the same way with temperature. In Sec. 3.4 we will see that the change in pressure or volume of all gases will yield the same temperature scale, if conditions are properly chosen. This *ideal gas temperature scale* is then used to calibrate thermometers which make use of the changes in other physical properties, such as the mercury-in-glass thermometer commonly used.

After the preceding discussion of the ideal gas temperature scale, the reader might wonder how very low and very high temperatures are measured. Helium may be used to measure temperatures down to about −269°C. Below that temperature helium is a liquid. However, the magnetic properties of certain substances vary systematically with temperature. Thus, the magnetic properties of one of these materials can be calibrated against the helium gas thermometer. At temperatures at which helium is a liquid, the measured value of the magnetism of the calibrated material can be used to provide a temperature scale by extrapolation. Similarly, at very high temperatures the glass used in a gas thermometer would melt. The electrical resistance

(See Sec. 4.2) of many materials varies regularly with temperature. If the variation in electrical resistance of a material is measured at moderate temperatures, the value of the electrical resistance at high temperatures can be used to calculate the temperature. Alternatively, the wavelength at which the emission from a very hot body is a maximum can be used to determine its temperature, as is discussed in Sec. 6.2.

3.2 Heat units and calorimetry

The flow of heat between bodies was studied before it was known that heat is a form of energy. (The relation between heat and other forms of energy will be discussed in Sec. 3.3.) For historical reasons, therefore, units of heat are defined in terms of the heat required to raise the temperature of a unit mass of water one degree on some temperature scale. The *calorie* is defined as the heat required to raise the temperature of one gram of water by one degree Celsius. The kilocalorie is defined as the heat required to raise the temperature of one kilogram of water by one degree Celsius. (The kilocalorie will be used throughout this book. It is also the unit used by nutritionists, who omit the prefix "kilo.") In the English-speaking countries the *British Thermal Unit* (BTU) is defined as the heat required to raise the temperature of one pound of water by one degree Fahrenheit. We will quickly note that 1 BTU equals 0.252 kilocalories, but will not use this unit again.

From the definition of the kilocalorie it is evident that if 2 kilocalories of heat are transferred to one kilogram of water, its temperature will change by 2°C. Similarly, if 2 kilocalories of heat are added to 4 kilograms of water, its temperature will only change by $\frac{2}{4} = 0.5$°C. Suppose, however, that the substance to which we add heat is not water. In general, something different than one kilocalorie will be needed to raise the temperature of one kilogram of the substance by one degree Celsius; this amount of heat is called the *specific heat* of the substance. In

different words, the specific heat of a substance is the ratio of the amount of heat required to raise the temperature of a given mass of the substance by a certain number of degrees to the heat required to raise the temperature of the same mass of water by the same number of degrees. Thus, specific heat is a pure number, without physical units, and has the same value for a given substance regardless of the heat unit and the temperature scale which might be used. Since the heat required to raise the temperature of a mass m of water by t°C is simply mt, if the heat H required to raise the temperature of the same mass of the substance through the same temperature, then from the definition of specific heat we can write:

$$s = \frac{H}{mt} \tag{36}$$

From Eq. 36 the heat required to raise the temperature of a mass m of a substance with specific heat s through a temperature interval Δt is given by:

$$H = ms\,\Delta t \tag{37}$$

The basic principle of *calorimetry* is that the heat gained by bodies which warm up equals the heat lost by bodies which cool down. Most laboratory work in calorimetry is done in an insulated container, such as a Thermos bottle shown in Fig. 33 so that heat is transferred only among a small number of substances, usually two or three. For each substance the heat it gains or loses is calculated from Eq. 37. The heats gained and lost are then equated and the resulting equation is solved for the unknown quantity, which is usually either the final temperature of the mixture or the specific heat of one of the substances. The examples given below will help the reader understand how to apply this principle.

Suppose that 2 kg of water at 80°C are mixed with 4 kg of water at 40°C. Let us call the final temperature of the mixture t. Then the change in temperature of the hot water is $(80 - t)$°C and the change in temperature of the cool water is $(t - 40)$°C.

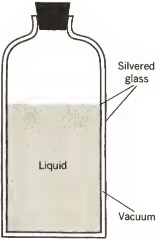


FIGURE 33

THERMOS BOTTLE
(Dewar flask)

If we use Eq. 34 to calculate the heats gained and lost and then equate these, we find:

$$2 \times 1 \times (80 - t) = 4 \times 1 \times (t - 40)$$

When we multiply out the parentheses in the above equation, we obtain:

$$160 - 2t = 4t - 160$$

The solution of the above equation yields $t = 53°\text{C}$.

As a second example of calorimetry, let us consider mixing together 2 kg of copper of specific heat 0.1 and temperature 200°C with 3 kg of water at a temperature of 20°C. As before we will call the final temperature of the mixture t. In this case the temperature of the copper decreases by $(200 - t)$°C and the temperature of the water increases by $(t - 20)$°C. If we use Eq. 37 to calculate the heats gained and lost, we find the following equation:

$$2 \times 0.1 \times (200 - t) = 3 \times 1 \times (t - 20)$$

After we multiply out the factors in the preceding equation we obtain:

$$40 - 0.2t = 3t - 60$$

The solution of the equation above is that $t = 100/3.2 = 31°\text{C}$.

3.3 The first law of thermodynamics

Some simple examples show us that various forms of energy can be converted into heat. If we sand a surface, both the surface and the sandpaper get hotter. Also, if we flex a piece of metal, such as a paper clip, it gets hotter. In each of these cases mechanical energy is converted into heat. In a less obvious way the cooking elements of an electric stove get hot because electrical energy is converted into heat. If we refer to Sec. 2.7, in which the conservation of mechanical energy was discussed, we might guess that we could extend this conservation principle to include heat and other forms of energy.

Speculations concerning the relation between heat and other forms of energy were made during the first part of the nineteenth century by Count Rumford, Helmholz, Mayer, and others. However, the quantitative experiments of James Prescott Joule, an English physicist in the period 1840–1860, gave the convincing proof of these ideas. Joule found that whenever a certain amount of any form of energy was converted into heat, the amount of heat produced equalled the amount of energy used. He converted mechanical, acoustical, and electrical energy into heat, always with a similar result. After two decades of experimentation he concluded that 4180 joules of any sort of energy would produce one kilocalorie of heat. Thus, we can treat heat as a form of energy on a par with other forms of energy. Naturally, in a given situation we must use the same unit for all of the energies appearing. That is, all energies may be measured in joules or all energies may be measured in kilocalories. To do this we use the experimental relation that 1 kilocalorie = 4180 joules, which is known as the *mechanical equivalent of heat.*

We can now generalize the *principle of conservation of energy* to include forms of energy other than the two mechanical types treated in Sec. 2.7. In a system isolated from the rest of the universe this principle states that the total of the various forms of energy in the system is conserved (remains constant). Naturally, one form of energy may be converted into another form,

but the sum of the various forms of energy in the system remains the same.

In the discussion earlier in this section of Joule's experiments on the conversion of various forms of energy into heat we have tacitly assumed that all of the energy added to the substance is converted into heat. This may not be true. For instance, the substance may expand and thus do mechanical work. Similarly, if we pass electrical energy through a battery, some of this energy will be converted into heat and some will be stored in the battery in the form of chemical energy. In these examples, not all of the energy added to a body is converted into heat. On the other hand, if heat energy is added to a body, some of this energy may be used to raise the temperature of the body and some may be converted into external work. If the work done by the body is less than the heat added to the body, we say that the difference has been used to raise the *internal energy* of the body.

We will put the ideas of the preceding paragraph on a more exact basis. Let an amount of heat H be added to a body and let the amount of work done by the body be W. Because of conservation of energy, W can never be greater than H. If W is less than H, we say that the internal energy U of the body has increased from an initial value U_1 to a final value U_2. The increase in internal energy of the body, $U_2 - U_1$, equals the difference between the heat H added to the body and the work W done by the body. This is known as the *first law of thermodynamics*, which can be expressed as an equation in the form:

$$\underset{\text{(added to body)}}{H} = \underset{\text{(done by body)}}{W} + \underset{\substack{\text{(increase in internal}\\ \text{energy of body)}}}{(U_2 - U_1)} \tag{38}$$

In Eq. 38 the reader should notice that two new forms of energy have been defined. Heat is recognized as a form of energy in transit, and the internal energy of the body is defined. This is then the extension of the principle of conservation of energy to include these new forms of energy.

The concept of energy and its conservation have been very useful in the development of science. When apparent violations of the principle occur or new phenomena are discovered, new forms of energy are introduced in order to make the principle continue to hold. For instance, in 1905 Einstein predicted theoretically that mass could be converted into energy. This was soon verified experimentally by nuclear physicists. The most striking example of this "mass-energy" is the atomic bomb. Only if we treat mass as a form of energy under certain conditions can we preserve the principle of conservation of energy in many nuclear experiments. It seems safe to say that in the future all necessary steps and inventions will be taken to maintain the energy balance intact.

3.4 Gases and the ideal gas temperature scale

We will now consider a confined gas. Let the force F act on an area A of the wall of the container. If the force had any portion parallel to the surface of the wall, by Newton's third law an equal and opposite force would be exerted by the wall of the container on the nearby gas. If this were so, the gas would move parallel to the wall of the container. Since such a motion is not observed if enough time has elapsed for the gas to be in equilibrium, we conclude that in equilibrium the force exerted by the gas on the wall of its container is at right angles to the surface of the wall. We define the *pressure* p exerted by the gas on the surface as the ratio of the magnitude of F to A. According to the third law of motion, this is also the pressure exerted on the gas. In the form of an equation we can write:

$$p = \frac{F}{A} \tag{39}$$

In the metric system, pressure is measured in the units of $\mathrm{nt/m^2}$, although other units, such as the $\mathrm{lb/in.^2}$, are commonly used.

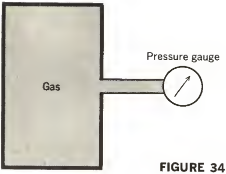


FIGURE 34

We live under a blanket of air, so that we experience an average pressure due to the weight of the atmosphere of about 15 $lb/in.^2$ or 10^5 nt/m^2. For many purposes it is more useful to know the difference between the actual pressure of a gas and the surrounding atmospheric pressure. This difference in pressure is known as the *gauge pressure.* For instance, the pressure in the tires of your car may be said to be 28 $lb/in.^2$ in terms of gauge pressure, while the *total or absolute pressure* in the tires would be about 43 $lb/in.^2$. In the remainder of this section pressure will always mean total pressure unless otherwise specified.

In Sec. 3.1 it was mentioned that the pressure of a confined gas changes with temperature. For simplicity, we will keep the gas in a rigid container as shown in Fig. 34 so that its volume cannot change. For any gas at sufficiently low pressure we find the relation between the pressure of the gas and its temperature in degrees Celsius to be as shown in Fig. 35. Furthermore, we

FIGURE 35

notice that for all gases at low pressure the straight line when extrapolated to the left would predict zero pressure for the gas at $-273°C$. (Actually, the gas would become a liquid at temperatures well above $-273°C$, but the remarks following apply as long as the substance is still a gas.) This behavior suggests that we define a new temperature scale with its zero at $-273°C$, so that the relation between the new scale T and the Celsius scale t, is as follows:

$$T = t + 273° \tag{40}$$

The size of the degree is the same in both scales, but their zeros are taken at different points. In terms of the new scale, the pressure of the gas p and its temperature T are seen to be proportional. If we introduce a constant of proportionality c we can write:

$$p = cT \text{ (constant volume)} \tag{41}$$

As another simple case we will next discuss the behavior of a gas at low pressure when the pressure is kept constant and the volume changes with temperature as shown in Fig. 36. For any gas we find the relation between volume and temperature at constant pressure to be as shown in Fig. 37. Amazingly, if we extrapolate the straight line to the left, we find that the volume would become zero at $-273°C$, if the substance were still a gas

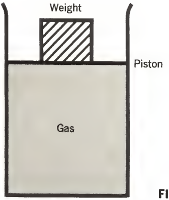


FIGURE 36

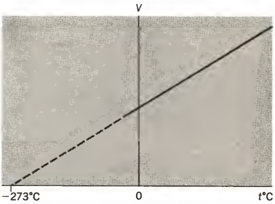


FIGURE 37

at such a low temperature. In this case too we find that we should define a new temperature T as given in Eq. 40. If we use this new temperature scale, we can say that the volume V of a gas is proportional to the temperature T. If we introduce a constant of proportionality c', we can write:

$$V = c'T \text{ (constant pressure)} \tag{42}$$

When we compare Eq. 41 with Eq. 42 we see that the same temperature scale is found experimentally for all gases at low pressure. A gas that obeys these two laws is called an ideal gas and the temperature scale defined using such a gas is given by Eq. 40. This temperature scale is known as the *ideal gas temperature scale* and is easily approximated very accurately in the laboratory using gases such as hydrogen or helium at low pressures. Thermometers based on this temperature scale are then used to calibrate such common thermometers as the mercury-in-glass thermometer.

A second look at Eq. 41 and 42 shows us that the two equations can be combined into a single equation. If we introduce a new constant of proportionality R, we can write

$$pV = RT \tag{43}$$

In order to obtain Eq. 41 from Eq. 43, we put $c = R/V$, while to get Eq. 42 from Eq. 43 we put $c' = R/p$. Thus, Eq. 43 describes

the behavior of ideal gases and is known as the *ideal gas law.* In solving problems involving ideal gases, usually we have an initial state given by p_1, V_1, and T_1, and a final state given by p_2, V_2, and T_2. If we solve Eq. 43 for R we can write:

$$R = \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{44}$$

Thus, we do not have to know the value of R in solving most problems. However, the reader should keep in mind that the temperature T must be the ideal gas temperature defined by Eq. 40 and the pressure p must be the total pressure on the gas.

The ideal gas law expressed in Eq. 43 can be derived on the basis of the *kinetic theory.* Here we assume that an ideal gas consists of molecules which occupy no volume and do not exert forces on one another except during the moment of impact during a collision. All of the molecules move in a wild, random motion which is continually changing as molecules collide.

From the preceding paragraph we might conclude that the irrational movements of the molecules would not let us make any calculations about them. This is not true, since the number of molecules involved is large and averages can be used. For instance, if a million coins are dumped on a pavement, we could not expect to predict whether a given coin would have a head or a tail on top, but we could predict fairly confidently that about half a million coins would have a head. In a similar way, kinetic theory does not let us predict what an individual molecule will do, but does allow us to calculate the effect of the entire collection.

Let us consider a gas molecule striking the wall of the container and rebounding, as is shown in Fig. 38. According to Eq. 23 of Sec. 2.8, the change in momentum of the molecule requires that the wall exert a force on the molecule. By Newton's third law, which is discussed in Sec. 2.5, the molecule must exert an equal and opposite force. Thus, the impact of the molecule exerts a small force on the wall of the container. If this idea is carried through mathematically for the case of N

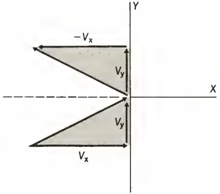


FIGURE 38

molecules, it is found that the pressure p, volume V, and average kinetic energy $\overline{\text{K.E.}}$ are related by the equation:

$$pV = \tfrac{2}{3}\mathcal{N}\overline{\text{K.E.}} \tag{45}$$

In the equation above we see that averaging over all of the $\mathcal{N}$ molecules has led to a definite result, even though individual molecules have different speeds and energies.

When we compare Eq. 45 with Eq. 43, we see that they can only be consistent if the following relation holds:

$$\begin{aligned} \overline{\text{K.E.}} &= \tfrac{3}{2}(R/\mathcal{N})T \\ &= \tfrac{3}{2}kT \end{aligned} \tag{46}$$

In Eq. 46 the constant $k = R/\mathcal{N}$ turns out to be a universal constant which is the same for all gases which are nearly ideal, such as hydrogen and helium. This constant is known as *Boltzmann's constant* and its value is $k = 1.38 \times 10^{-23}$ joules per °K. (The absolute or Kelvin temperature scale is defined in Sec. 3.6 and corresponds to the ideal gas temperature defined by Eq. 40.)

As an example of the use of Eq. 46, let us compute the average energy of a molecule of a gas at 27°C. This temperature corresponds to $T = 27 + 273 = 300$°K. We have then:

$$\overline{\text{K.E.}} = \tfrac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.20 \times 10^{-21} \text{ joules}$$

While this may seem like a small amount of energy, the kinetic energy of all the molecules in one ounce of air at this temperature turns out to be about 3670 joules = 900 calories. If the

atmospheric pressure has its standard value of about 10^5 nt/m^2, the average speed of an air molecule under these conditions will be about 510 m/sec = 1140 mph. For comparison, it should be noted that the energy of an air molecule under these conditions is about $\frac{1}{30}$ as much as the energy released when oxygen and hydrogen combine to form a water molecule and about 3×10^{-11} as much as the energy released when a single uranium nucleus undergoes fission. (See Sec. 7.5 for a discussion of nuclear fission.)

In the case of real gases, the simple assumptions of the kinetic theory of ideal gases evidently are inadequate. Molecules do exert forces on one another as they approach, and molecules do occupy volume. When these factors are taken into account, as was first done by Van der Waals, a Dutch physicist, Eq. 43 takes the modified form given below:

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT \tag{47}$$

In the equation above, a and b are constants characteristic of the gas being studied. The constant a corrects the observed pressure for the attractions between molecules and the constant b corrects the observed volume for the volume occupied by the molecules. This equation describes real gases quite well, and even more refined equations do better.

As we discussed above, individual molecules of a gas have different speeds. The distribution in speeds for an ideal gas was first calculated by a Scottish physicist, James Clerk Maxwell, about a century ago. A typical distribution in molecular speeds for a gas at a given temperature is shown in Fig. 39. It should be noted in the figure that many molecules have speeds greatly differing from the average or most probable speed. Since a light molecule, such as hydrogen, has a greater speed at a given temperature than a heavy molecule, such as oxygen, the earth's gravitational attraction is sufficient to hold oxygen in our atmosphere, while hydrogen escapes the earth's gravitational pull. This helps to explain why there is little hydrogen in our atmosphere, even though it is one of the most common elements.

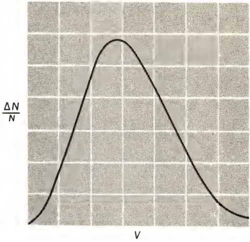


FIGURE 39

If the temperature of the gas is raised, the whole distribution curve moves to the right on the diagram and so does the most probable speed. When the theory is carried through quantitatively, we find that a relatively small increase in temperature (proportional to the average energy per molecule) greatly increases the number of molecules with speeds exceeding a given value. If the temperature of our earth rose only slightly, the oxygen and nitrogen in the atmosphere would soon eave our planet because the molecular velocities would enable molecules to escape the earth's gravitational influence. Fortunately, our source of heat, the sun, seems to have been quite constant for a very long time.

The kinetic theory can be verified directly in a simple way discovered by a Scottish botanist, Robert Brown, in the early nineteenth century. He found that small particles suspended in a liquid and viewed with a microscope moved about erratically. This *Brownian movement* can be easily viewed in air today by observing smoke from a cigarette illuminated transversely under a microscope. If we watch a given particle, we find that it moves about in a quite unpredictable manner. The particles we are talking about are very small (perhaps 10^{-4} cm) but they are still large compared to the size of the liquid molecules. By chance,

one side or another of the suspended particle may receive more impacts by the liquid molecules and thus the particle will receive an impulse in a particular direction. A moment later, another side of the particle may receive the majority of the impacts, so that the particle will move in another direction. This accounts for the erratic motion. Albert Einstein in 1905 proposed a theory of this motion which was verified by J. B. Perrin, a French physicist and chemist, several years later. Their combined work was quite important in establishing the atomic theory and determining the mass of atoms. Officially, at least, this was one of the scientific contributions for which Einstein received the Nobel prize in physics, rather than for his theories of relativity, which are now more famous. By about 1915 this and other experiments had pretty well convinced the scientific world of the existence of atoms and molecules.

*3.5 Properties of gases, liquids, and solids

In the preceding section we discussed the properties of the ideal gas, which can be well approximated experimentally by using a gas at low pressure. If we study a real gas we find that as we subject the gas to higher pressures and lower temperatures, the gas will condense into a liquid. Similarly, as we lower the temperature of a liquid it eventually becomes a solid. In this section we will consider the relations between these three states of matter and how they depend on such influences as pressure and temperature.

Consider some liquid which partially fills a closed container, as shown in Fig. 40. At a given temperature the liquid will evaporate until above the liquid we find a constant vapor pressure. This is known as the *equilibrium vapor pressure* of this particular liquid at this temperature. If we raise the pressure on the vapor or lower the temperature of the combination, some of the vapor will condense into liquid. If we measure the equilibrium vapor pressure p of the vapor of a given liquid at various temperatures T, we will find a curve similar to that shown in

FIGURE 40

Fig. 41. Consider the point marked A in Fig. 41. For this particular pair of values of pressure and temperature the liquid and its vapor can exist in equilibrium in any proportions. However, if we raise the pressure, all of the substance will become a liquid. On the other hand, if we raise the temperature, all of the substance becomes a vapor. Thus, the *equilibrium vapor pressure curve* shown in Fig. 41 separates the region of liquid from the region of vapor. This is known as a *phase-diagram.*

From the discussion of the preceding paragraph we might guess that we could always change a vapor into a liquid at any temperature by applying a large enough pressure. This is not true, however. At temperatures above a certain temperature,

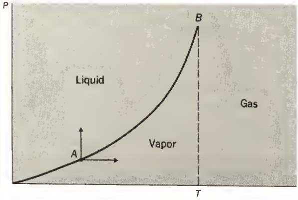


FIGURE 41

known as the *critical temperature*, no amount of pressure will change the substance from the gaseous state into the liquid state. In Fig. 41 the critical temperature is shown as the point marked B. It should be noted here that it is customary to call a substance a gas when it is at a temperature above its critical temperature and a vapor when it is at a temperature below its critical temperature. Clearly, for a substance to approximate an ideal gas it must be at a temperature well above its critical temperature.

Similarly we could investigate the relation between pressures and temperatures at which a solid and its liquid could exist together in equilibrium in any proportions. A typical equilibrium curve for melting or freezing is shown in Fig. 42. At the point marked C in Fig. 42 the solid and liquid exist together in equilibrium. If we now raise the pressure or lower the temperature, all of the substance will become a solid. (Here it should be noted that water is unusual in that its melting curve slopes to the left rather than to the right, so that ice melts under high pressure.) Thus, the *melting or fusion curve* on a phase diagram separates the region in which all of the substance is a solid from the region in which all of the substance is a liquid.

In some cases a solid changes directly into a vapor. Common examples are the evaporation of mothballs and dry ice (frozen carbon dioxide). This process is known as *sublimation*. As we

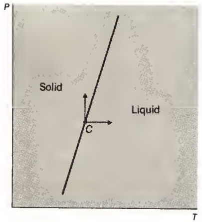


FIGURE 42

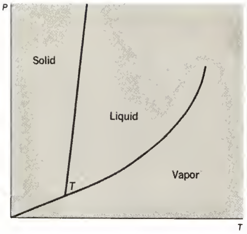


FIGURE 43

have seen earlier in this section, there are combinations of pressure and temperature at which the solid and its vapor can exist together in equilibrium in any proportions whatsoever. If we make a graph of these combinations, we obtain the *sublimation curve* of the substance.

We might now think it worthwhile to plot the vapor pressure, fusion, and sublimation curves all on a single graph of pressure against temperature. Unfortunately, the changes in pressure and temperature which are required to show all three of these curves are very large, so that it is not possible to show all of them drawn to correct scale on one diagram. Therefore, Fig. 43 shows the three curves of changes of state in a qualitative way. The most striking feature of Fig. 43 is that there is a particular combination of pressure and temperature marked T on the diagram at which the vapor, liquid, and solid can exist together in equilibrium in any proportions. This is known as the *triple point.*

It is well known that a given mass of ice has a greater cooling effect on a drink than the same mass of ice water. This is because the ice absorbs heat as it melts and then the resulting ice water cools the drink more. The heat required to melt a unit mass of a substance without change in its temperature is known as the *heat of fusion* of the substance. For the case of ice the heat of fusion is about 80 kilocalories per kilogram. Similarly, if we let

rubbing alcohol evaporate from our skin, the skin is cooled. Evidently, heat is required to change the liquid alcohol into vapor, and this heat is supplied by our skin. The heat required to change a unit mass of a liquid into vapor is called the substance's *heat of vaporization*. For water the heat of vaporization depends on temperature and is about 540 kilocalories per kilogram at 100°C. The *heat of sublimation* of a substance is similarly defined as the heat required to change a unit mass of the substance into vapor.

As an example of the energy involved in change of phase, let us consider adding 100 kilocalories of heat to a piece of ice having a mass of 2 kg at a temperature of 0°C. We find that the amount of ice melted is $100/80 = 1.25$ kg. The result is that we now have 1.25 kg of water and 0.75 kg of ice, both at a temperature of 0°C. If we begin again with the same piece of ice, but add 200 kilocalories of heat to it, we will use up 160 kilocalories in melting the ice and obtaining 2 kg of water at 0°C. The remaining 40 kilocalories will raise the temperature of the water by $40/2 = 20$ degrees. Thus, the final temperature of the water will be 20°C.

As an additional example of the relation between temperature and heat added to water when changes of phase are involved, we will consider adding heat at a constant rate to 1 kg of ice initially at 0°C. The temperature of the material is shown as a function of the heat added in Fig. 44. From the figure we see that a total

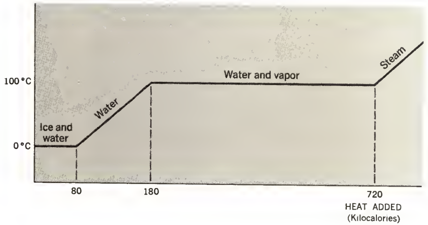


FIGURE 44

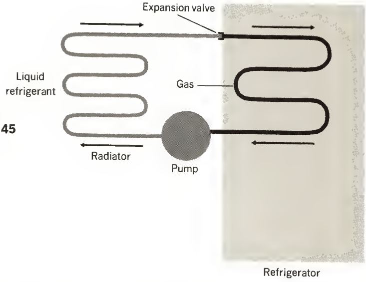


FIGURE 45

of 720 kilocalories of heat must be added to change 1 kg of ice at 0°C into 1 kg of steam at 100°C.

Conservation of energy tells us that when a liquid freezes it must give up heat per unit mass equal to its heat of fusion. Thus, when one kilogram of water freezes, 80 kilocalories of heat are given up. This is often useful, as this heat may keep the temperature of the surroundings from dropping below 0°C. When a vapor condenses into a liquid, heat equal to the substance's heat of vaporization is given up. This is used in steam radiators, where each kilogram of steam gives up 540 kilocalories of heat. Finally, when snow is formed from the change of water vapor into small ice crystals, the heat of sublimation is given up, thus warming the air somewhat.

The energy changes described in the preceding two paragraphs are used in the common household refrigerator to transfer heat from the food inside the refrigerator to the air of the room. Suppose that we have a liquid, such as Freon, which vaporizes readily at room temperature. When some of this liquid evaporates in the cooling coils of the refrigerator, as shown in Fig. 45, the heat of vaporization of the liquid is absorbed from the food, thus cooling the food. The warm vapor

is now compressed by the pump into a liquid and the heated liquid is cooled by the air of the room passing over the cooling fins. The liquid is then ready to be vaporized again and to cool the food further. The net result of this cyclic process is to transfer heat from the food to the room, leaving the liquid basically unchanged. The whole cost of the process is involved in operating the pump which transforms the vapor into a liquid. In the following section we will study more carefully the energy balance in a cyclic process of this sort, since this is an example of a heat engine.

There are various ways in which heat can be transferred from one body to another. In the process known as *convection* heat is gained by a material; the heated material physically moves and then warms a colder object. An extreme example of convection would be taking a warm brick to bed on a cold night, so as to warm your feet. In the case of hot-air furnaces, the heating unit in the furnace warms the air. Since this warm air is less dense than the cool air in the house, it rises, heating the house. Heat may also be transferred by *conduction*, in which the material does not actually move. In this case, the heat energy is passed along from one atom or molecule to its neighbor in somewhat the same way that one bowling pin knocks down the next one. A familiar example of heating by conduction is the electric stove, in which the heating element transfers heat to the pan and then the pan transfers heat to the food. The third way in which heat energy can be transferred is by *radiation*. As we will see in Chapters 5 and 6, waves transmit energy through a medium without affecting the medium and heat is carried by waves similar to light waves. The most common example of this sort of transfer of heat is the transmission of energy from the sun to the earth by means of waves. Here it should be noted that the transmission of energy by radiation does not require the presence of a material medium. The reader is referred to Sec. 6.8 for a more detailed discussion of this idea.

When the temperature of a substance is changed, usually its physical dimensions also change. For nearly all substances an increase in temperature produces an increase in size, which

is known as *thermal expansion*. We notice this in an ordinary thermometer in which an expansion of the liquid causes it to rise inside the glass tube. Bridges have expansion joints to allow for the contraction and expansion of the bridge during the seasons.

If we make measurements on the change in length, ΔL, of a solid bar when the temperature changes by an amount ΔT, to a good approximation we find that the change in length is proportional both to the change in temperature and the initial length of the bar L_0. If we introduce a constant of proportionality a, we can then write:

$$\Delta L = aL_0 \, \Delta T \tag{48}$$

The *coefficient of linear expansion*, a, is therefore the fractional change in length, $\Delta L/L_0$, per unit change in temperature. Typical values for metals are in the range of 10^{-5} per °C.

As an example of the use of Eq. 48, let us consider the change in length of a steel rail initially 100 feet long and at a temperature of 20°C. For steel $a = 1.2 \times 10^{-5}/°\text{C}$ approximately. If the temperature increases to 30°C, the increase in the length of the rail is given by:

$$\Delta L = 1.2 \times 10^{-5} \times 100 \times 10 = 1.2 \times 10^{-2} \text{ ft} = 0.144 \text{ in.}$$

Thus, the change in length is not insignificant. If the rail is not free to expand this much, according to Hooke's law, which we discussed in Sec. 2.11, it will exert a very large force on the rails at its two ends. Usually, therefore, railroad tracks are laid with small gaps between successive rails.

An example of a material which contracts when it is heated is rubber. If you hang a small weight with a rubber band and then gently heat the rubber with a match flame, you will see the weight rise as the rubber contracts. Water also contracts as its temperature is raised from 0°C to 4°C. Thus, water near its freezing temperature is less dense than slightly warmer water. As a pond begins to freeze, the denser and warmer water sinks to the bottom, while the coldest water is at the top. Therefore, ice forms at the top of the pond rather than at the bottom. Since

water expands on freezing, ice is less dense than water, so the pond freezes from the top down. Ice is not a very good conductor of heat, so that usually a pond will not freeze all the way to the bottom. If this were not true, fish could not survive a cold winter.

Changes in the volume of a substance are also associated with changes in temperature. If the change in volume is ΔV and the change in temperature is ΔT, it is found that ΔV is approximately proportional to ΔT and the original volume V_0. If we introduce as a constant of proportionality the *coefficient of volume expansion* b, we can write then:

$$\Delta V = bV_0\,\Delta T \tag{49}$$

In words, b is the fractional change in volume, $\Delta V/V_0$, per degree change in temperature ΔT. If the material is isotropic (has the same properties in all directions), it is found that approximately $b = 3a$.

Suppose that a liquid has a coefficient of volume expansion of 10^{-3} per °C, which is typical. If the original volume of the liquid in a tank was 100,000 gallons at a temperature of 10°C, when the temperature rises to 30°C the increase in volume would be

$$\Delta V = 10^{-3} \times 10^{5} \times 20 = 2000 \text{ gallons}$$

In this example, the volume increases by 2 percent. For this reason, when liquids are stored in tanks, the tanks are never filled to the top, so as to allow for the increase in volume of the liquid as the temperature rises.

As a final example of the expansion of materials with temperature we will consider *shrink-fitting*. Suppose that we wish to secure a circular collar on a cylindrical rod. If we make the collar slightly too small, it will not fit on the rod. However, if we heat the collar, it will expand enough so that it will slip over the rod. After the collar cools down, it will then fit very tightly on the rod. This process is used quite a bit in industrial applications. It will be left to the reader to consider how the collar could be removed once it has been put on in the way described above.

3.6 Heat engines and the second law of thermodynamics

In Sec. 3.3 we discussed the conversion of a given amount of some form of energy into heat energy. We also extended the principle of conservation of energy to include heat and the internal energy of a substance as forms of energy. In this section we will see that it is never possible to convert all of a given amount of heat energy into some other form of energy while keeping the internal energy of the substance the same. We will begin this topic by considering the properties of heat engines.

In its simplest form a *heat engine* consists of a device which takes in heat H_1 at a high temperature T_1, does some external work W, and exhausts some heat H_2 at a low temperature T_2. The operation of a heat engine is shown schematically in Fig. 46. The working substance in the engine is taken through a series of values of pressure, temperature, and other variables. When the conditions of the working substance have been returned to the initial values of pressure, temperature, and so on, we say that we have taken the substance through a *cycle*. All theoretical

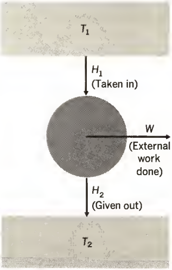


FIGURE 46

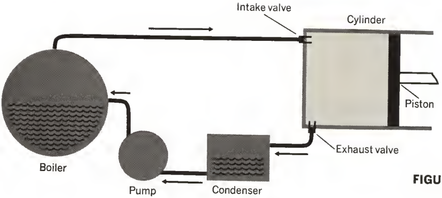


FIGURE 47

discussions of heat engines involve taking the working substance through a complete cycle, so that the condition of the substance is unchanged at the end of the cycle.

As an example of a practical heat engine we will consider a simple steam engine. As is shown in Fig. 47, water is converted into steam in the boiler by the addition of heat, which corresponds to the heat H_1 in Fig. 46. The steam expands in the cylinder and pushes the piston, which does mechanical work corresponding to W in Fig. 46. The steam then condenses into water and gives up an amount of heat corresponding to H_2 in Fig. 46. The water has now been taken through a complete cycle and is ready to enter the boiler where a new cycle will begin.

Since the state of the working substance of a heat engine is the same at the beginning and end of a cycle, the internal energy of the working substance is also unchanged. Thus, in Eq. 38 $U_1 = U_2$. In Sec. 3.3 we treated the heat added to a body as positive and the heat given up by a body as negative. For the heat engine shown in Fig. 45, Eq. 38 takes the form:

$$H_1 - H_2 = W \tag{50}$$

Here it should be pointed out that not all of the work W is useful because of losses within the engine. The work W is simply the amount of energy which has been converted from heat energy into some other form, whether useful or not.

A useful term is the *efficiency* of a heat engine, which we define as the ratio of the work output of the engine to the total energy input to the engine. In the form of an equation we can write for the efficiency e of an engine the equation:

$$e = \frac{W}{H_1} \tag{51}$$

When we substitute the value of W from Eq. 50 into Eq. 51 we find:

$$e = \frac{H_1 - H_2}{H_1} = 1 - \frac{H_2}{H_1} \tag{52}$$

Thus we can only make the efficiency of conversion of heat 100 percent if the amount of heat, H_2, given out by the engine is zero.

While it is possible to convert all of some forms of energy into heat, attempts to do the reverse have failed. Evidently, any heat engine must give out some heat, so that its efficiency can never be 100 percent. Heat flows through the engine and some of the heat energy is converted to other forms of energy. A somewhat similar situation is found in the case of the water wheel, which takes some of the energy from the flowing water and converts it into mechanical energy. The impossibility of converting all of a given amount of heat into other forms was summarized by Lord Kelvin and Max Planck during the nineteenth century in the statement:

No engine acting in a cycle can take in a given amount of heat and convert it entirely into other forms of energy.

This is one statement of the *second law of thermodynamics.*

A *reversible engine* is an engine which runs backward with exactly the same properties if the value of one of the variables, such as pressure or temperature, is changed slightly. In practice there is no such thing as a reversible engine, because of energy losses due to friction and other factors. Thus, a reversible engine is an ideal heat engine which can at best be only approximated. Early in the nineteenth century Nicolas Carnot, a

French physicist, showed that all reversible heat engines operating between the same temperatures, T_1 and T_2, must have the same efficiency. He showed further that the efficiency of any other engine had to be less than the efficiency of a reversible engine operating between the same pair of temperatures. Of course, this is not surprising, as all ideal engines should have the same properties in a general sense, including their efficiencies, and ideal engines should be more efficient than non-ideal engines. Even though it is impossible in practice to build an ideal engine, the theoretical properties of ideal engines set an upper limit on the performance of real engines, so that the concept is a useful one.

From Eq. 52 we see that the efficiency of a heat engine depends on the amounts of heat taken in and given out during a cycle. Since Carnot had shown that the efficiencies of all reversible (ideal) engines operating between the same intake and exhaust temperatures is the same, it is evident that the efficiency of a reversible engine depends on the temperatures at which it takes in and gives out heat, T_1 and T_2, and on no other factors. Thus, the efficiency of a reversible engine is a property of nature which depends on temperature and can be used to construct a temperature scale.

Kelvin established the *Kelvin or absolute temperature scale* by stating that the temperature ratio $T_2/T_1 = H_2/H_1$. From Eq. 52 we see that the efficiency of a reversible engine in terms of this temperature scale is given by:

$$e = 1 - \frac{T_2}{T_1} \tag{53}$$

Kelvin also showed that if an ideal gas was used as the working substance in the heat engine, the ideal gas temperature defined in Sec. 3.4 was identical to the absolute temperature defined by Eq. 53. Thus, while no ideal engine can be constructed, the absolute temperature scale can be approximated as closely as we please by using a gas at very low pressure. (Other substances can also be used if they are similarly restricted.) The ideal gas temperature scale and the absolute temperature scale are

identical, and temperatures on these scales are expressed in degrees Kelvin (°K). We have then:

$$T(°\mathrm{K}) = 273 + t(°\mathrm{C}) \qquad \textbf{(54)}$$

In all theoretical work in thermodynamics the Kelvin scale is used, although many measurements are made using the Celsius scale.

As an example, let us compute the ideal efficiency of a heat engine operating between the temperature of 127°C and 27°C. The corresponding Kelvin temperatures are found from Eq. 54 to be 400°K and 300°K. The efficiency of this eal engine is then:

$$e = 1 - \tfrac{300}{400} = 0.25 = 25 \text{ percent}$$

A real engine operating between this pair of temperatures would have a lower efficiency because of losses, but at least we have an upper limit on the engine's possible efficiency. Since the lower temperature T_2 is usually approximately room temperature (20°C), the efficiency of a real engine is best increased by increasing the temperature T_1 at which heat is taken into the engine. For instance, steam-engines are operated at high pressures so that the temperature of the steam will be much higher than 100°C. The other way in which we can increase the efficiency of a real engine is to reduce the losses; this is also attempted.

*3.7 Entropy and the second law of thermodynamics

In the preceding section we stated the second law of thermodynamics in terms of the negative principle that no heat engine could convert all of the heat energy which it takes in into some other form of energy. We will now introduce the concept of entropy and use this idea to state the second law of thermodynamics in a different way. The formulation of the law in terms of entropy is more mathematical and is thus better suited to theoretical studies in thermodynamics.

Suppose that an amount of heat H is added to a heat reservoir at a constant temperature T. (In order for the temperature of the reservoir to remain constant as heat is added to it, the reservoir must have a very large heat capacity. The ocean would serve very well for this purpose.) We define the *increase in entropy* S of the reservoir by the equation:

$$S = \frac{H}{T} \tag{55}$$

Similarly if a reservoir at a temperature T gives out an amount of heat H, the *decrease in entropy* of the reservoir is given by $-H/T$. As we did in Sec. 3.3, heat added to a body is considered positive and heat given out by a body is considered negative. Here we should note that if the reservoir does not have a large heat capacity, then the amount of heat flow must be sufficiently small for the temperature of the reservoir not to change appreciably if we wish to use Eq. 55 to calculate the change in entropy.

Let us now apply the concept of entropy to the ideal heat engine discussed in Sec. 3.6 and shown diagrammatically in Fig. 45. Since the heat H_1 flows out of the hot reservoir at the temperature T_1, the entropy of this reservoir decreases by an amount $-H_1/T_1$. Also, the cold reservoir gains an amount of heat H_2 at a temperature T_2, so that its entropy increases by an amount H_2/T_2. Since these are the only heat flows in the universe in this particular situation, the change in entropy of the universe is given by:

$$S = \frac{H_2}{T_2} - \frac{H_1}{T_1} \tag{56}$$

The engine is an ideal engine, so the definition of absolute temperature tells us that $T_2/T_1 = H_2/H_1$. If we use this relation in Eq. 56, we find that the entropy change of the universe during the operation of an ideal heat engine acting through a cycle is zero.

Consider a non-ideal (actual) heat engine operating between the temperatures T_1 and T_2 and taking in the same amount of

heat H_1 at the temperature T_1 as the ideal engine discussed in the preceding paragraph. According to Carnot's theorem stated in Sec. 3.6, the non-ideal engine has a lower efficiency than the ideal engine. Therefore, for a given heat intake H_1 it does less work and gives out more heat H_2 at the lower temperature T_2. In Eq. 56 every quantity is the same on the right side for the ideal and the non-ideal engines, except H_2, which is larger for the non-ideal engine. Thus, S is positive for the non-ideal engine and the entropy of the universe increases when heat flows through a non-ideal engine.

From the preceding discussion we see that when heat flows in cyclic processes, the entropy of the universe is either unchanged or increases. A way of stating the second law of thermodynamics is as follows:

In cyclic processes involving heat flow the entropy of the universe never decreases.

This statement can be shown to be entirely equivalent to the Kelvin-Planck statement of the same law, but it has the advantage of being stated in terms of a mathematically defined quantity, entropy. For actual, non-ideal substances there is always an increase in entropy in any process. This leads to an accumulation of heat energy at a low temperature where it is largely unavailable. Thus, entropy measures the unavailability of energy.

SUMMARY

Any discussion of heat or thermodynamics must be based on a temperature scale. We began by defining the Fahrenheit and Celsius (or Centigrade) scales and considering the relationship between them. Later we discussed the temperature scale based on the properties of a hypothetical ideal gas. For this ideal gas, the three variables

of temperature, pressure, and volume become zero at $-273°C$, so the new scale was defined as having its zero at that temperature with the degrees being equal to those on the Celsius scale. Finally we discovered that the Kelvin or absolute temperature scale, based on the properties of an ideal heat engine rather than on those of any real substance, is identical to the ideal gas temperature scale.

Traditionally the flow of heat between bodies is described by units such as the kilocalorie, which is defined as the heat required to raise the temperature of one kilogram of water by one degree Celsius. After defining the commonly used heat units, we considered the processes involved in heat flow and in particular noted that the heat gained by one body must equal that lost by the other. As heat was shown to be a form of energy, the principle just stated is simply an example of conservation of energy. Since all heat added to a body is not always converted into work, we defined a new type of energy—the internal energy of the body. This allowed us to apply the principle of conservation of energy and derive the first law of thermodynamics: The heat added to a body equals the work done by the body plus its increase in internal energy.

Gases are of interest in any discussion of thermodynamics for reasons other than the use of an ideal gas to define a practical temperature scale. It was shown that the experimentally observed relations between pressure, temperature, and volume of ideal gases can be derived on the basis of the kinetic theory which treats gas molecules as small particles moving in a random way. This led to a study of the energy changes involved in going from one of the three states of matter—gaseous, liquid, or solid—to another. It also led to a discussion of thermal expansion. The mechanism by which an ordinary household refrigerator cools the food inside it was used as an example of these energy changes.

A consideration of heat engines, which convert heat to other forms of energy, led us to the second law of thermodynamics, which states that it is impossible to convert all of a given amount of heat into some other form of energy. This idea was put into more exact form by defining changes in entropy and stating that in cyclic processes the entropy of the universe never decreases or, in other words, heat processes can lead only to more energy becoming unavailable.

4 60 20 / 0 24

PROBLEMS

1 *Compute the value of 98.6°F on the Celsius scale.* ANS.: *37.0°C.*

2 *Compute the value of 0°C on the Fahrenheit scale.*

3 *Determine the temperature at which the Fahrenheit and Celsius scales have the same value.* ANS.: *−40.*

4 *The volume of a gas kept at constant pressure increases from 4 cubic meters (m^3) at 0°C to 5 m^3 at 100°C. Find the temperature at which the volume of this gas would be 4.65 m^3.*

5 *Compute the heat required to raise the temperature of 5 kg of water by 15°C.* ANS.: *75 kilocalories.*

6 *The specific heat of lead is 0.03. Calculate the amount of heat necessary to raise the temperature of 20 kg of lead by 30°C.*

7 *Repeat Problem 6 for aluminum, which has specific heat 0.20.* ANS.: *120 kilocalories.*

8 *A kilogram of water falls a distance of 20 m. If it is assumed that all of the mechanical energy due to falling this distance is converted into heat, compute the rise in temperature of the water.*

9 *Five kilocalories of heat are added to a gas which does 10^4 joules of work. Compute the change in internal energy of the gas. Is it an increase or decrease?* ANS.: *1.09×10^4 joules; increase.*

10 *How much heat must be added to a body if it does 20,000 joules of work and its internal energy increases by 30,000 joules?*

11 *Two kilocalories of heat are added to a body and its internal energy decreases by 10^4 joules. How much work is done by the body or on the body in this process?* ANS.: *18,360 joules of work done by the body.*

12 *Suppose that each of your shoes has an area of 10 in.^{2} and your weight is 180 lb. Compute the pressure exerted on your shoe-soles by the ground.*

13 *A gas is originally at an absolute pressure of 10^5 nt/m^2 when its volume is 2 m^3 and its temperature is 27°C. The temperature of the gas is raised to 87°C and its volume increases to 2.5 m^3. Compute the pressure of the gas under these conditions.* ANS.: *9.6×10^4 nt/m^2.*

14 *The tires in your car have a gauge pressure of 30 lb/in.^{2} on a day when the atmospheric pressure is 15 lb/in.^{2} and the temperature is 20°C. After a long drive the temperature of the tires has risen to 60°C, while the volume of the tires has not changed appreciably. Compute the gauge pressure in the tires now.*

15 *Explain how a pressure cooker works, using Fig. 41.*

16 *The fusion curve of water slopes to the left instead of sloping to the right as shown in Fig. 42 for most substances. Explain what effect this might have on ice-skating.*

17 *One-tenth kilogram of ice at 0°C is added to 2 kg of water at 80°C. Compute the final temperature of the mixture.* ANS.: *72.5°C.*

18 *Compute the heat given off when 3 kg of steam at 100°C condenses into water and then cools down to 60°C.*

19 *An inventor claims that his heat engine takes in 5 kilocalories of heat, gives off 2 kilocalories of heat, and does 15,000 joules of work. What is the efficiency of the engine? Would you invest money in this engine?* ANS.: *71.6%; no.*

20 *An ideal engine is to take in 3 kilocalories of heat and do 10^4 joules of work. How much heat is exhausted by the engine and what is its efficiency?*

21 *Refer to Problem 20. If the intake temperature is 127°C, what must its exhaust temperature be?* ANS.: *−192°C.*

22 *Two kilograms of ice melts and gives off 160 kilocalories of heat at a constant temperature of 0°C. Compute the change in entropy in this situation. Does this violate the second law of thermodynamics?*

23 *A heat engine takes in 4 kilocalories of heat at a temperature of 227°C, does 5000 joules of work, and exhausts heat at a temperature of 27°C. Compute the efficiency of this engine and the amount of heat exhausted.* ANS.: *29.9%; 2.81 kilocalories.*

24 *Refer to Problem 23. Compute the change in entropy in this process.*

25 *Compute the value of 80°F on the Celsius scale.* ANS.: *26.7°C.*

26 *Compute the value of 27°C on the Fahrenheit scale.*

27 *Compute the temperature at which the Fahrenheit value is twice the Celsius value.* ANS.: $C = 160°$; $F = 320°$.

28 *The volume of a gas kept at constant pressure is 6 cubic meters at 100°C. Compute the temperature at which this gas would have a volume of 4.20 cubic meters.*

29 *Compute the heat required to raise the temperature of 3 kg of water from 20°C to 25°C.* ANS.: *15 kilocalories.*

30 *When 20 kilocalories of heat are added to a certain amount of water, the temperature of the water is raised by 4 degrees Celsius. Compute the mass of the water.*

31 *The specific heat of copper is 0.093. Calculate the amount of heat required to raise the temperature of 8 kg of copper from 20°C to 25°C.* ANS.: *3.72 kilocalories.*

32 *Compute the distance which water must fall vertically so that the temperature of the water will be raised by 0.1°C on striking the ground. Assume that half of the mechanical energy dissipated is converted into heat.*

33 *Eight kilocalories of heat are added to a gas which does 20,000 joules of external work. Compute the change in internal energy of the gas and state whether this is an increase or decrease.* ANS.: *13,400 joules increase.*

34 *Compute the amount of heat in kilocalories which must be added to a gas which does 30,000 joules of external work and has its internal energy decrease by 10,000 joules.*

35 *A car weighs 4000 lb and each of its tires is inflated to a gauge pressure of 25 lb/in.² Compute the area of each tire which is in contact with a horizontal roadway when the car is at rest.* ANS.: *40 in.²*

36 *A gas is at a pressure of one atmosphere when its volume is 3 m³ and its temperature is 27°C. If the pressure on the gas is raised to 3 atmospheres and its temperature rises to 97°C, compute its volume under these conditions.*

37 *A gas has an initial pressure of 10^4 nt/m², initial volume of 3 m³, and initial temperature of 20°C. If the gas is compressed so that its pressure is 10^5 nt/m² and volume is 1 m³, compute its temperature.* ANS.: *704°C.*

38 *At a temperature of 27°C and a total pressure of 30 lb/in.², a certain gas has a density of 10^{-3} gm/cm³. Compute the density of the gas when its temperature is raised to 77°C and its pressure raised to 75 lb/in.²*

39 *How much ice at 0°C must be added to 0.5 kg of water at a temperature of 40°C so that the temperature of the resulting mixture will be 10°C?*
ANS.: *0.167 kg.*

40 *If 0.1 kg of ice is added to 0.3 kg of water at a temperature of 20°C, what is the temperature of the resulting mixture?*

41 *Compute the heat required to convert 2 kg of ice at 0°C into steam at 100°C.*
ANS.: *1440 kilocalories.*

42 *If 0.1 kg of steam condenses on 2 kg of aluminum at a temperature of 20°C and specific heat 0.20, compute the final temperature of the aluminum.*

43 *If the exhaust temperature of an ideal engine is 27°C, what must be its intake temperature if the efficiency of the engine is to be 20 percent?*
ANS.: *102°C.*

44 *A steam engine takes in heat at 500°F and exhausts it at 70°F. If the efficiency of this engine is 20 percent of the efficiency of an ideal engine, compute the efficiency of the steam engine.*

45 *An ideal engine takes 5 kilocalories of heat and does 10^4 joules of external work. How much heat is exhausted by the engine and what is its efficiency?*
ANS.: *2.60 kilocalories; 48%.*

46 *Refer to Problem 45. If the exhaust temperature of the engine is 27°C, what is its intake temperature?*

47 *An ideal engine operates between the temperatures of 300°C and 20°C. If it is to do 10^4 joules of external work, how much heat does it take in?*
ANS.: *4.92 kilocalories.*

48 *Refer to Problem 47. How much heat does this engine exhaust?*

49 *An ideal engine exhausts heat at a temperature of 20°C. Compute its intake temperature if its efficiency is to be 90 percent.* ANS.: *2657°C.*

50 *A heat engine takes in 5 kilocalories of heat at a temperature of 337°C and exhausts 3 kilocalories of heat at a temperature of 27°C. Compute the amount of external work done by this engine and its efficiency.*

51 *Refer to Problem 50. Compute the change in entropy in this process.*
ANS.: *0.00181 kilocalories/°K.*

52 *Take the average mass of a molecule of air to be 4.76×10^{-23} gm. Compute the average speed of such a molecule at a temperature of 27°C.*

53 *Take the coefficient of thermal expansion of steel to be 1.2×10^{-5} per degree Celsius. Compute the change in length of a bridge which is 6000 ft long at 20°F when the temperature increases to 100°F.*
ANS.: *3.2 ft.*

DISCUSSION QUESTIONS

1 *How would you suggest measuring the temperature of inaccessible objects, such as the sun and the upper atmosphere?*

2 *When a mercury-in-glass thermometer is dipped into hot water, the mercury column first falls and then rises. Why?*

3 *Why should a substance like rubber contract when it is heated, while most solids expand?*

4 *Water has the uncommon property of contracting as its temperature is raised from 0°C to about 4°C, and then expanding as its temperature is raised further. Use this information to explain why ice-skating is possible on the surface of a deep lake.*

5 *Explain the difference between the scientific usages of the words "temperature" and "heat."*

6 *Can heat be added to a substance without raising its temperature?*

7 *Discuss the most important requirements for a material out of which you hope to make an excellent frying pan.*

8 *When a thin piece of metal is flexed repeatedly, its temperature rises. Why is this?*

9 *As a gas molecule moves around a room its gravitational potential energy changes. Do you think that this energy change is significant? Justify your answer quantitatively.*

10 *Although the average energy of molecules in a gas at a certain temperature is the same, the molecules do not all have the same speed. Explain this apparent paradox.*

11 *Give one or more examples of the conversion of some form of energy into heat energy.*

12 *Could you cool your kitchen on a hot day by leaving your refrigerator door open?*

13 *Is there a change in entropy when a ball is thrown up into the air and then caught?*

14 *Suppose that the temperature of a given amount of gas is raised a certain number of degrees, first while the pressure of the gas is kept constant and secondly while the volume of the gas is kept constant. In which process would more heat be required?*

15 *In a* Carnot cycle *the working substance takes in an amount of heat* H_1 *at a constant temperature* T_1, *and then does work without any further heat transfer. The cycle continues with the substance giving off heat* H_2 *at a constant temperature* T_2, *and the cycle is completed by having work done without heat transfer in such a way that the substance is brought back to its initial state of pressure, volume, temperature, and so on. If the substance is a gas, sketch this cycle on a diagram on which pressure is plotted against volume.*

16 *Refer to the preceding question. Sketch the same cycle on a diagram on which temperature is plotted against entropy.*

17 *It is said of a heat pump used to heat houses that more heat is delivered to the house than is taken from the ground. Does this statement violate any of the laws of thermodynamics?*

18 *Are there heat transfers which violate the second law of thermodynamics? If so, explain the apparent paradox.*

19 *Show that for an isotropic substance the coefficient of volume expansion should be three times the coefficient of linear expansion for the same substance.*

20 *Most substances expand when they are heated. Hang a weight with a rubber band and gently heat the rubber band with a match.*

21 *The energy in a room full of air is quite large. Explain why we cannot use this energy to heat our houses.*

22 *Basically, temperature is defined only for substances in thermal equilibrium. Explain how kinetic theory could be used to define a temperature for particles in the interior of the sun.*

23 *Does the kinetic theory apply to an object such as a car?*

CHAPTER FOUR

ELECTRODYNAMICS

4.1 Electrostatics

Electrostatic effects are quite common in everyday life. After we have walked across a rug in dry weather, we often get a shock and notice a small spark when we touch a metal object, such as a doorknob. If we comb our hair, we find that the comb will pick up small pieces of lint and paper. Under certain conditions an enormous spark, which we call a lightning bolt, passes between a cloud and the earth. While apparently different, all of the examples above are cases of electrification by friction, which we will discuss quantitatively in the remainder of this section.

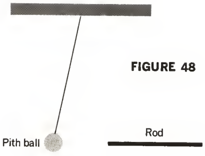


FIGURE 48

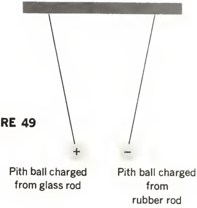


FIGURE 49

Suppose that we rub a hard rubber rod with a piece of wool cloth and then touch the rod to a small piece of light wood, such as pith or balsa wood, suspended by a thread. The small ball is then repelled by the rubber rod, as shown in Fig. 48. Presumably something has been transferred from the rubber rod to the small ball which causes this force of repulsion. If we rub a glass rod with a silk cloth and then touch it to another small and light ball, the ball will be repelled by the glass rod. Again we assume that something has been transferred from the glass rod which causes this repulsion. If now we bring the two small balls close together, they attract one another, as shown in Fig. 49. Let us call the substance which has been transferred from either rod to the ball *electrical charge.* From the experiments described above, we see that a charge will repel a similar charge but will attract a dissimilar charge. From this we conclude that there must be two types of electrical charge. Experiments have failed to discover a third type of charge. The two types of charge could have been labelled A and B, but in the late eighteenth century Benjamin Franklin suggested that the charge appearing

on the glass rod be called positive (+) and the charge appearing on the rubber rod be called negative (−). This definition is still in use today.

In 1787 C. A. Coulomb, a French physicist, measured the forces between small charges, using apparatus similar to that described in Sec. 2.9. He found that for a given pair of charges the force was quadrupled when the distance between them was halved, while if the distance between the charges was tripled, the force between them was reduced to $\frac{1}{9}$ of its original value. If we consider two charges of magnitudes Q_1 and Q_2, the force between them is also found to be proportional to the product Q_1Q_2. We can summarize both of these observations by saying that the force between charges Q_1 and Q_2 separated a distance d is proportional of Q_1Q_2/d^2. The situation is shown in Fig. 50 for two charges of the same sign. If we introduce a constant of proportionality k, we can then write:

$$F = k\,\frac{Q_1Q_2}{d^2} \tag{57}$$

We notice in Eq. 57 that if we halve the distance d, the force is increased by a factor of four, provided that the charges remain the same. Also, if the distance of separation is kept constant, we see from Eq. 57 that the force between the charges is proportional to the product of the charges. Thus, Eq. 57 correctly gives the results of Coulomb's observations. The direction of the force will be an attraction if the charges are of different types, while the force will be a repulsion if the two charges are of the same type.

In order to calculate the value of the constant k in Eq. 57 we must measure the force between a pair of known charges which are a measured distance apart. The constant k is thus an experimental constant. Up to this point, we have a unit for distance (the meter) and a unit for force (the newton). We now need a unit for charge. Since in practical work the effects of moving charges (electrical currents) are much more important than electrostatic effects, the fundamental unit in electricity is the *ampere*, which describes the rate of flow of charge per unit

of time. (The ampere is defined in Sec. 4.4.) The ampere is analogous to the rate of flow of a liquid through a pipe, which might be measured in gallons per second. If a constant current I measured in amperes flows for a time of t seconds, the total electrical charge passing a point Q is then given in coulombs by the expression:

$$Q = It \tag{58}$$

Thus, the unit of charge which we will use is the *coulomb* as defined by Eq. 58. Now that we have a unit in which to measure charges, we can determine experimentally the value of the constant k in Eq. 57 as explained earlier in this paragraph. The value of k turns out to be very nearly $k = 9 \times 10^9$, when the charges are measured in coulombs, their separation in meters, and the force between them in newtons.

As an example, let us compute the force between charges of 4×10^{-6} coulombs (4 microcoulombs) and -2×10^{-6} coulombs separated a distance of 0.1 meter. From Eq. 57, we have for this force the value:

$$F = 9 \times 10^9 \frac{(4 \times 10^{-6})(2 \times 10^{-6})}{0.1^2} = 9 \times 10^9 \frac{8 \times 10^{-12}}{10^{-2}} = 7.2 \text{ nt}$$

Thus we find that in this example the force between the given charges is 7.2 newtons and is an attraction, since the charges are unlike. If the separation of the charges is now increased to 0.2 meters, it is easily seen that the force between them is reduced by the factor four, so that the force now becomes 1.8 newtons.

In the discussion above one charge exerts a force on the other across a distance d. Philosophers during the nineteenth century did not like this idea of action-at-a-distance, so another concept was introduced. Let us consider the force on Q_2 as shown in Fig. 50. We say that the charge Q_1 establishes an *electric field* E at the location of the charge Q_2. This electric field

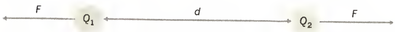
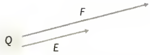


FIGURE 50



F = maximum and directed into page **FIGURE 51**

then exerts a force on Q_2. If the force on Q_2 is F, we *define* the electric field acting on Q_2 by the equation:

$$E = \frac{F}{Q_2} \tag{59}$$

From Eq. 59 it is evident that the units of electric field are newtons per coulomb. More generally, we find the electric field at any point in space by observing the force F on a positive charge Q placed at that point, as is shown in Fig. 51. It is understood in the definition above that the test charge Q is small enough so as not to disturb the very situation which we wish to measure.

A convenient way of describing electric fields pictorially is by means of *lines of force.* At each point in space the electric field has a definite direction and magnitude. Lines of force are drawn in such a way that at each point the direction of the line of force gives the direction of the electric field. Furthermore, the spacing between lines of force is chosen so that the number of lines of force at right angles to a unit area perpendicular to the diagram is proportional to the magnitude of the electric field. Both of these ideas are shown in Fig. 52. Clearly, such a diagram

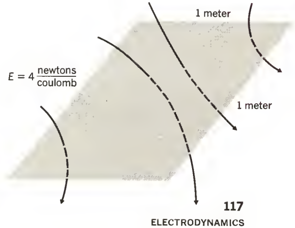


FIGURE 52

only gives an approximate value of the magnitude and direction of the electric field at any given point.

Let us return to the situation shown in Fig. 50, in which a charge Q_1 exerts a force F on a charge Q_2 when they are separated a distance d. From Eq. 57 we find that the force on Q_2 is $k(Q_1Q_2/d^2)$. When we substitute this force into Eq. 59, which defines the electric field on Q_2, we find:

$$E = \frac{F}{Q_2} = \frac{k(Q_1Q_2/d^2)}{Q_2} = k\frac{Q_1}{d^2} \tag{60}$$

Since the magnitude of the test charge Q_2 cancelled from Eq. 60, we could have made it as small as we pleased. Thus, Q_2 would not disturb the electric field which we were trying to calculate, and the electric field produced by any charge Q_1 at a distance d from it is given by Eq. 60. If several charges all produce electric fields at a given point, their contributions must be added vectorially using the method described in Sec. 2.4.

Let us compute the electric field produced at a distance of 0.2 meters from a point charge of 4×10^{-6} coulombs. From Eq. 60 we have then:

$$E = 9 \times 10^9 \frac{4 \times 10^{-6}}{0.2^2}$$

$$= 9 \times 10^9 \frac{4 \times 10^{-6}}{4 \times 10^{-2}} = 9 \times 10^5 \text{ nt/coul}$$

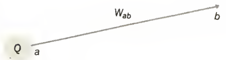
If now a charge of 5×10^{-6} coulombs is placed at this point, the force on it is given by:

$$F = (5 \times 10^{-6})(9 \times 10^5) = 4.5 \text{ nt}$$

In this case it would have been simpler to calculate the force on the charge by the direct use of Eq. 57, without bothering to calculate the electric field produced by the first charge.

In the static case it makes no difference whether we use Coulomb's law to find the force exerted on a given charge or break the problem into two parts in which we first calculate the electric field at the location of the charge and then find the force exerted on the charge by the electric field. If the charge or

FIGURE 53



charges which are establishing the electric field are moving, however, we *must* use the electric field idea. The reason for this is that changes in the force exerted by one moving charge on another are propagated with the speed of light (186,000 miles per second). If either the charge exerting the force or the charge experiencing the force has a speed close to the speed of light, changes are not transmitted instantaneously.

Since a charge located in an electric field experiences a force, it requires work to move the charge from one point to another if a force must be exerted against the force of the electric field to accomplish the change. Similarly, if the electric field is allowed to move the charge about, work is done by the electric field which would usually appear as an increase in the kinetic energy of the charged particle. Suppose that a certain amount of external work W_{ab} must be supplied to move a particle of charge Q from point a to point b as shown in Fig. 53. We *define* the *potential difference* between the points a and b, V_{ab}, by the equation:

$$V_{ab} = \frac{W_{ab}}{Q} \qquad \textbf{(61)}$$

In the system of units used in this book potential difference is expressed in joules per coulomb. Since potential difference is a very useful concept, its unit is given a name, the *volt,* after Count Alessandro Volta. Thus, one volt represents a potential difference of one joule per coulomb. If we have to do external work against the electric field force in order to accomplish the change in position of the charge, we have increased the electrostatic potential energy of the charge Q. By a suitable device we could later recover this work. Similarly, if the electric field pushes the charge Q to its new position, we can extract work during the process and the charge Q loses electrostatic potential energy.

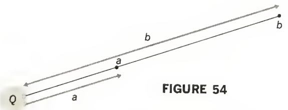


FIGURE 54

Suppose that it requires 12 joules of external work to move a charge of 2 coulombs from terminal a to terminal b of a battery. The potential difference between the terminals of the battery is then given by $V_{ab} = 12/2 = 6$ volts. If the battery were allowed to move the same charge from terminal b to terminal a, the work done on the charge would be 12 joules. When we say that the potential difference available in our household electrical wiring is 110 volts, we mean that 110 joules of work are done on each coulomb of electricity which is moved from one terminal of an outlet to the other. This amount of work is converted into some form of energy, such as heat or light, by the appliance connected between the two terminals of the electric outlet.

We might next wonder about the potential difference between a distance a from a charge Q to a greater distance b from this charge. The situation is shown in Fig. 54. The calculation involves mathematics beyond the scope of this book, but the result is as follows:

$$V_{ab} = k\left(\frac{Q}{b} - \frac{Q}{a}\right) \tag{62}$$

Since b is greater than a, V_{ab} is negative, which means that we can extract work from the system by letting a positive charge move from a to b.

4.2 Steady electric currents

As we mentioned in the preceding section, most practical applications of electricity involve moving charges. In some important cases the charges are accelerated, as in a radio antenna, but we

will consider only charges moving with constant velocity. We will also restrict our discussion to materials in which charges can move easily, which are known as *conductors*. Metals are usually conductors, with copper and silver being the best. Since copper is cheaper than silver, it is undoubtedly the material used in the electrical wiring in your house.

The rate of flow of electrical charge past a given point per unit of time is called the *electrical current* or more simply the current. The MKS unit of current is the ampere, which is defined in Sec. 4.4. Other units of current in common use are the milliampere (10^{-3} ampere) and the microampere (10^{-6} ampere). As examples from everyday life we might mention that a 100-watt electric light bulb carries a current of about one ampere, an electric stove may use a current of 20 amperes, and a hearing aid might carry a current of 10 milliamperes.

When we apply a potential difference between the ends of a conductor (perhaps by using a battery), work is done on the charges in the conductor and forces are exerted on these charges. We would expect that the greater the potential difference applied, the greater the force on each charge. Thus, we would assume that a greater applied potential difference would lead to a more rapid flow of a charge and consequently to a greater current. Apparently, the connection between the potential difference applied between the ends of a conductor and the resulting current was first investigated by Cavendish about 1800. However, he did not publish his results, so that the relation between these two quantities is named after the German physicist, G. S. Ohm. Ohm found (about 1840) that for metals (good conductors) the current I was proportional to the applied potential difference V. If we introduce a constant of proportionality R, we can write *Ohm's law* in the form:

$$V = RI \tag{63}$$

R is found to be almost exactly a constant for pure metals and is called the *electrical resistance,* or more simply the resistance, of the conductor on which the measurements are made.

When we solve Eq. 63 for R we find:

$$R = \frac{V}{I} \tag{64}$$

Since in our system of units potential difference is measured in volts and current in amperes, the unit of resistance is the volt per ampere, which is called the ohm. Here it should be pointed out that many electrical devices, such as vacuum tubes, transistors, motors, and so forth, do not obey Ohm's law. Those devices which do obey Ohm's law are often called *ohmic.*

Suppose that a potential difference V is applied between the ends of a conductor and a current I flows through the conductor. During a time t a charge Q flows through the conductor, according to Eq. 58. The work done by the source of potential difference is then given by Eq. 61 to be:

$$W = QV = (It)V \tag{65}$$

If we divide both sides of Eq. 65 by the time t, we obtain the work per unit time, which we defined in Sec. 2.7 to be the power delivered. We have then:

$$P = \frac{W}{t} = IV \tag{66}$$

The relation given in Eq. 66 does not depend on the conductor obeying Ohm's law, since it comes directly from the definitions of work, power, potential difference, and current. It holds therefore for *any* electrical device which conducts a current I when a potential difference V is applied across it. Since current is measured in coulombs per second and potential difference is measured in joules per coulomb, we find that the power is expressed in watts, as shown below:

$$P = IV = (\text{coul/sec}) \times (\text{joules/coul}) = \text{joules/sec} = \text{watts}$$

If the conductor obeys Ohm's law, so that it is said to be ohmic, we can combine Eq. 63 with Eq. 66 to get two additional

expressions for the power used in a conductor. If we substitute the value of V from Eq. 63 into Eq. 66, we find:

$$P = I^2R \tag{67}$$

Since $I = V/R$ from Eq. 63, if we substitute this value for I into Eq. 66 we get:

$$P = \frac{V^2}{R} \tag{68}$$

The reader should note at this point that Eq. 66 applies to any conductor of electricity, good or bad, ohmic or not, while Eq. 67 and Eq. 68 apply only to conductors which obey Ohm's law.

As an example, let us consider an electric light bulb which uses 100 watts when connected across 110 volts. From Eq. 66 we have that the current conducted by this light bulb is given by:

$$I = \frac{P}{V} = \frac{100}{110} = 0.909 \text{ amp}$$

If we assume that the filament inside the bulb obeys Ohm's law, we can write:

$$R = \frac{V}{I} = \frac{110}{0.909} = 121 \text{ ohms}$$

We should note that the resistance of the filament could just as well have been obtained from Eq. 68 if we had written:

$$R = \frac{V^2}{P} = \frac{110^2}{100} = 121 \text{ ohms}$$

Suppose that an automobile battery delivers 150 amperes at a potential difference of 12 volts when the car is being started. Under these conditions the power used is given by:

$$P = VI = 12 \times 150 = 1800 \text{ watts}$$

In order for this to be possible, the resistance of the starter-motor and the rest of the circuit connected to the battery must have a resistance given by:

$$R = \frac{V}{I} = \frac{12}{150} = 0.0800 \text{ ohms}$$

We see from this example that a poor connection at one of the terminals of the battery can keep the battery from delivering enough current and power to start the engine of the car.

The bills which electrical utility companies send their customers are usually expressed in terms of kilowatt-hours (KWH). We define the kilowatt-hour as the amount of work done when energy is delivered at the rate of one kilowatt (1000 watts) for one hour. Thus, we see that one kilowatt-hour $=$ 1000 watt-hours $=$ 3,600,000 watt-seconds $=$ 3,600,000 joules. This very large amount of work or energy is delivered to the consumer at a price of approximately 5 cents, depending on his location. If we run a 100-watt bulb for 5 hours, we use up $0.1 \times 5 = 0.5$ KWH of electrical energy, which would cost us several cents. Similarly, if an electric clock draws 2 watts of power, in a month of 30 days it would use an amount of electrical energy given by $2 \times (24 \times 30)/1000 = 1.44$ KWH, so that operating this clock would cost about 7¢. On the other hand, if the broiler of our oven uses 2000 watts and we roast a turkey for 5 hours, we would use $2000 \times 5/1000 = 10$ KWH, so that the cost would be about 50¢. In all of these cases, however, the cost of the electrical energy used is usually very small as compared to the cost of other sources of energy.

4.3 Magnetostatics

Before the time of Christ it had been observed that there are forces between certain pieces of black rock commonly found in Magnesia in Asia Minor. This mineral is called magnetite and small pieces of it are known as *magnets.* Aside from the forces between pieces of magnetite, it was also found that magnetite would attract small pieces of iron, such as nails. Furthermore, this effect is different from electrification by friction, which was discussed in Sec. 4.1, since no previous rubbing of the piece of magnetite was needed, and a magnet had no effect on a small piece of lint or paper. Materials which respond to magnetite are

said to be magnetic and this area of physics is called *magnetism.*

If a long, thin piece of magnetite is mounted on a cork and the cork is floated on water, the cork will rotate until the piece of magnetite lines up in approximately the north-south direction. This was the basis for the earliest *compasses,* which were used for navigational purposes probably by the tenth or eleventh century. From this effect we conclude that the earth exerts a force on a magnet, so that the earth itself must be a large magnet. The end of the piece of magnetite which points northward is called the north pole of the magnet, while the end which points southward is called its south pole.

A more careful examination of a magnet shows that the magnetic effect in a long, thin piece of magnetite is mainly found at or near its ends. These ends are called its *magnetic poles* and are called north or south poles, depending on which geographical pole of the earth the particular end of a piece of magnetite points to when it is free to rotate or act as a compass. Furthermore, like poles repel one another and unlike poles attract. In 1789 Coulomb measured the forces between magnetic poles. If we let p_1 and p_2 be the pole strengths of two magnetic poles separated a distance d, Coulomb found that the force between the poles was proportional to p_1p_2/d^2. The reader should note that the force between magnetic poles is of the same form as the gravitational force between masses described in Sec. 2.9 and the electrostatic force between charges discussed in Sec. 4.1. In fact the apparatus that Coulomb used to observe the electric and magnetic forces was very similar to that used by Cavendish to measure the gravitational force.

Here we should note that the earth acts magnetically as if it had a large magnet at its center. The origin of this magnetism is not known, but it is suggestive that the density of the inner part of earth is approximately the same as that of iron and nickel, which are known to be magnetic substances. If we treat the interior of the earth as a large bar magnet, we find that its axis is not parallel to the axis of rotation of the earth. Thus, the north and south magnetic poles do not coincide with the north and south geographic poles of the earth.

By analogy with the electrostatic case treated in Sec. 4.1, we might expect that we could replace the idea of action-at-a-distance between two magnetic poles by a field concept. We would then say that one magnetic pole establishes a *magnetic field* at the location of the second magnetic pole, and this magnetic field exerts a force on the second pole. Until fairly recently this was commonly done, but it is now plain that the effects we call magnetic are due to moving charges and are thus not really basic. However, the idea of magnetic fields is still used, as will be explained in the following paragraph.

Suppose that we have a motionless charge near a magnet. Although the magnet presumably sets up a magnetic field, no force is exerted on the charge unless the charge is moving with a velocity. Since this force is not caused by any electrostatic effects, we call it a magnetic force, and we say that the magnetic field exerts a force on the charge moving through it. If we change the direction of the motion of the charge, the magnitude of the force on the charge changes. We can always find a particular direction of the motion of the charge in which the force on the charge is zero, as is shown in Fig. 55. We define the *direction* of the magnetic field to be this direction. If we now let the charge move in a direction at right angles to the direction in which it experienced zero force, the force on the charge will become a maximum, which we will call $F_{\max}$. This situation is shown in

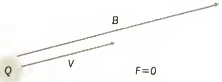


FIGURE 55

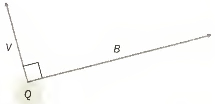


FIGURE 56

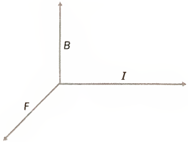


FIGURE 57

Fig. 56. If the magnitude of the charge is Q and its speed is v, we define the *magnitude* of the magnetic field, B, by the equation:

$$B = \frac{F_{\max}}{Qv} \tag{69}$$

In the MKS system of units used in this book, force is measured in newtons, charge in coulombs, and speed in meters per second. From this we see that the unit of *magnetic field strength* (or intensity) is 1 nt/coul-m/sec. Because of the importance of magnetic fields, this unit is given a name—the weber per square meter—after a German physicist, and we have then:

$$1 \text{ weber}/\text{m}^2 = 1 \text{ nt}/\text{coul-m}/\text{sec}$$

In order to give the reader some idea about the magnitudes of magnetic fields, the magnetic field of the earth is about 5×10^{-5} webers/m^2, while an electromagnet may produce a magnetic field of 1 weber/m^2 or greater.

Since an electric current is simply a group of moving charges, a magnetic field will exert a force on a current. For a current I flowing in a straight conductor of length L at right angles to a magnetic field of strength B as shown in Fig. 57, it can be shown that the force on the conductor is given by:

$$F = BLI \tag{70}$$

In Eq. 70 we must measure the magnetic field in webers/m^2, the length of the conductor in meters, and the current in amperes. Then the force on the conductor comes out in newtons.

As an example, let us assume that a short circuit in an electric power station produces a momentary current of 10^4 amperes in a conductor 2 meters long at a point where the earth's magnetic field is 5×10^{-5} webers/m² and at right angles to the conductor. From Eq. 70, the force on the conductor is given by:

$$F = 5 \times 10^{-5} \times 2 \times 10^4 = 1 \text{ newton}$$

If the mass of the conductor happened to be 0.5 kg, then the conductor would experience an acceleration given by:

$$a = \frac{F}{m} = \frac{1}{0.5} = 2 \text{ m/sec}^2$$

Since the acceleration of gravity is about 9.8 m/sec², this is not a negligible acceleration nor is the magnetic force exerted on the conductor small compared to the force of gravity. In cases of short circuits, the magnetic forces may actually rip conductors from their mountings. Here it should be pointed out that forces of the same sort are used in electrical motors.

4.4 Magnetic effects of currents

In the preceding section we discussed the action of magnetic fields on moving charges and their equivalent, electric currents. According to Newton's third law, if a magnet exerts a force on an electric current (by the magnet's establishing a magnetic field), then an equal and opposite force must be exerted on the magnet by the electric current. If we use the field idea, the force on the magnet must be due to a magnetic field acting at the location of the magnet. The only explanation we can give, which is consistent with these concepts, is that an electric current must set up a magnetic field which then exerts a force on the magnet.

The ideas expressed in the paragraph above were first observed experimentally by Oersted, Biot, and Savart during the early nineteenth century. They found that an electric current did in fact produce a magnetic field. In addition, this magnetic

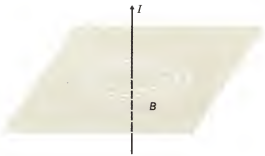


FIGURE 58

field had the same effect on a magnet or another conductor carrying an electric current as would the magnetic field set up by a magnet. For the case of a long, straight conductor carrying a current I, they found that at a distance d at right angles to the conductor the current established a magnetic field B given by:

$$B = 2 \times 10^{-7} \frac{I}{d} \tag{71}$$

The magnetic field of a long, straight conductor is shown in Fig. 58. In Eq. 71 the magnetic field strength is given in webers/m^2 when the current is measured in amperes, and the perpendicular distance from the conductor to the point in question is measured in meters.

In Sec. 4.3 we calculated the force on a conductor which had the large current of 10^4 amperes. Now let us compute the magnetic field produced by such a current at a perpendicular distance of 1 cm from the conductor. From Eq. 71 we have:

$$B = 2 \times 10^{-7} \frac{10^4}{10^{-2}} = 0.2 \text{ webers/m}^2$$

Thus, a parallel conductor of length l meter carrying a current of 10^4 amperes also would experience, according to Eq. 70, a force of

$$F = 0.2 \times 1 \times 10^4 = 2 \times 10^3 \text{ nt}$$

This force is almost certainly very large compared to the weight of the conductor. Thus, if both conductors carry such a large current due to a short circuit, the magnetic force exerted by one

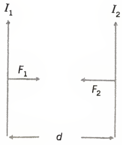


FIGURE 59

conductor on the other may be sufficient to pull one or the other of the conductors from its mountings.

The numerical example of the paragraph above suggests that we calculate the force per unit length between a pair of long, parallel conductors separated a distance d. Let the conductors carry currents I_1 and I_2 as shown in Fig. 59. Then the magnetic field produced by the current I_1 at the location of current I_2 is given by:

$$B_1 = 2 \times 10^{-7} \frac{I_1}{d} \tag{72}$$

Combining Eq. 72 with Eq. 70, we find for the force per unit length on the current I_2 the expression:

$$F_2 \text{ (per unit length)} = B_1 I_2 = 2 \times 10^{-7} \frac{I_1 I_2}{d} \tag{73}$$

If the two currents, I_1 and I_2, are the same, and are equal, say, to I, Eq. 73 takes the form:

$$F \text{ (per unit length)} = 2 \times 10^{-7} \frac{I^2}{d} \tag{74}$$

Eq. 74 provides the definition of the ampere, from which the coulomb and other electromagnetic units are defined. We define the *ampere* as that current which, when flowing in long,

parallel conductors spaced a distance of 1 meter apart, causes a force of 2×10^{-7} newtons per meter of their length between them. As is often the case, practical measurements of currents measured in amperes use indirect means derived from Eq. 74.

As an example of the use of Eq. 73, let us calculate the force per meter between two long, parallel conductors spaced 1 cm apart when each carries a current of 10^4 amperes. We have then:

$$F \text{ (per meter)} = 2 \times 10^{-7} \frac{10^4 \times 10^4}{10^{-2}} = 2 \times 10^3 \text{ nt}$$

The result above agrees with the earlier calculation in this section, in which we first found the magnetic field produced by one conductor at the location of a second conductor and then found the force exerted on the second conductor by this magnetic field. Naturally, the two different calculations concerning the same situation must give the same result.

Since we have seen in this section that electric currents (moving electric charges) produce magnetic fields, we might speculate that all magnetic effects are caused by moving electric charges. This idea was proposed by A. M. Ampere, a French physicist, in the first half of the nineteenth century, but it could not be put into quantitative form until the theory of atomic structure was developed during the first part of the twentieth century. According to the atomic theory, atoms consist of heavy, positively charged nuclei about which move various negatively charged particles, called electrons. In addition to their motions about nuclei, the electrons were also thought of as clouds of electric charge which rotated about their axes. This rotational motion of electric charge then leads to the explanation of the magnetic properties of iron, magnetite (an iron oxide), and other materials. For a number of years it has been evident that the properties which we call magnetic are caused by the motions of electric charges in the atoms which make up all matter. Today, magnetism is no longer considered a separate branch of physics, but rather as an aspect of *electrodynamics,* which deals with both motionless and moving electric charges.

4.5 Induced electromotive force

In the preceding section we saw that moving electric charges (electric currents) produced magnetic fields and thus have an effect on motionless magnets. Quite often we find in nature reciprocal effects. We might therefore guess that a moving magnet might have some sort of effect on a charge at rest by producing an electric field. In this section we will see that this guess is correct, and that it can be generalized.

In about 1840, Michael Faraday noticed that a current would flow in the coil shown in Fig. 60 during the time that the nearby magnet was being moved about. A few years before this an American, Joseph Henry, had observed that when a current is changed in one coil, a momentary current was produced in a second coil near the first. The experimental arrangement is shown in Fig. 61. Looking back from our present vantage point we see that the common feature in these two experiments is that in each case a current is produced in a coil whenever the coil is subjected to a *changing* magnetic field. In Faraday's experiment the motion of the magnet made the magnetic field at the location of the coil change, while Henry's observations can be explained by noting that the varying current in the first coil produced a changing magnetic field in the vicinity of the second coil. Although Henry published his results earlier than Faraday did, at the time the United States commanded very little attention from the world's scientists, so that this discovery of the relation between a changing magnetic field and its effect on charges is credited to Faraday, an Englishman. Before continuing the discussion of these experiments, we will digress somewhat.

FIGURE 60

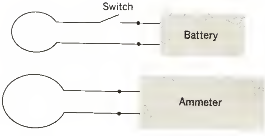


FIGURE 61

As we discussed in Sec. 4.1, work is required to move a charge about. We defined the potential difference between two points as the work needed to move a unit positive charge from one point to the other. Algebraically, the definition of potential difference (or voltage) is given by Eq. 61. If we do work to move a charge through a conductor, the work is converted into heat and other forms of energy, such as mechanical energy. According to the second law of thermodynamics treated in Sec. 3.6, not all of the heat energy thus produced can be converted back into electrical or other forms of energy. We therefore say such a process is *irreversible.* In some cases, at least ideally, no heat is involved and we can completely transform electrical energy into some other form (or forms). Furthermore, we can transform the other forms of energy entirely into electrical energy, at least in principle, if heat energy plays no part. Processes in which a mutual conversion is possible between electrical energy and some other form of energy, are said to be *reversible.* Because of such effects as electrical heating and mechanical friction, no process in nature is truly reversible, but we can often approach very closely this ideal situation.

When a device produces a potential difference between its terminals and is also reversible, we say that the device is a source of *electromotive force*, which is often abbreviated "emf." (The reader should note that the word "force" is not used correctly, since the device must be able to transform reversibly work or energy, not force. However, this misuse of the word is the common practice.) If a device transforms an amount of energy W into electrical energy when a charge Q passes through the

device, or if the device transforms an amount of electrical energy W into some other form of energy (not heat energy) when a charge Q passes through the device, and if the device is reversible, we say that the device is a source of electromotive force $\mathcal{E}$ of magnitude given by

$$\mathcal{E} = \frac{W}{Q} \tag{75}$$

If the work or energy is measured in joules and the charge is measured in coulombs, then the magnitude of the electromotive force is measured in volts. We see from the definition that a source of electromotive force always produces a potential difference, but a potential difference is not always due to a source of electromotive force.

Perhaps the simplest example of a source of electromotive force is the battery which we use in our car. When the battery is used to start the car, chemical energy in the battery is converted (almost) reversibly into electrical energy, which is then converted into mechanical work by the starter motor. Once the car is running, mechanical energy from the motor operates a generator, which produces electrical energy. This electrical energy is then converted into chemical energy and stored in the battery, so that the battery is ready to start the car another time. As a second example of a source of electromotive force, we might consider the motor or generator. When electrical energy is supplied to a motor, it converts this energy into mechanical energy. In reverse, if we supply mechanical energy to the same device, we can produce electrical energy, as takes place in a generator. In both examples given above, the processes are not completely reversible because some of the energy is converted into heat by electrical resistance or mechanical friction. Nevertheless, the concept of a reversible device can be very useful.

Although the experiments of Faraday and Henry seem superficially different, they can both be explained in the same way if we introduce one more idea. Suppose that we have an area A in a plane and the area is bounded by a conductor. This is shown in Fig. 62. Let the magnetic field B in this region be

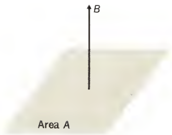


FIGURE 62

uniform and at right angles to the area. Then we define the *magnetic flux* f through the area bounded by the conductor by the equation:

$$f = BA \tag{76}$$

Since the magnetic field is measured in webers/m^2 and the area in m^2, magnetic flux is measured in webers. Although it will not concern us in this book, if the magnetic field is not at right angles to the area bounded by the conductor, we use the component (part) of the magnetic field which is at right angles to the area in calculating the flux from Eq. 76.

As an example, the vertical part of the earth's magnetic field is about 5×10^{-5} webers/m^2. A typical house of rectangular shape might be 5 by 10 meters, so that it has an area of 50 m^2. The magnetic flux through a horizontal floor of this house would then be given by:

$$f = (5 \times 10^{-5})50 = 2.5 \times 10^{-3} \text{ webers}$$

Similarly, a large electromagnet might produce a magnetic field of 1.5 webers/m^2 throughout an area of 20 m^2, so that the total magnetic flux through this area would be

$$f = 1.5 \times 20 = 30 \text{ webers}$$

Now that we have defined electromotive force and magnetic flux, we can return to the discussion of the effect of a changing magnetic situation on electric charges. Suppose that the magnetic flux through a plane area A bounded by a conductor

changes from f_1 to f_2 during a time t. Then we can say that there is induced (produced) in the conductor an average electromotive force $\mathcal{E}$ given by:

$$\mathcal{E} = \frac{f_1 - f_2}{t} \tag{77}$$

In Faraday's experiment the magnetic flux changed because the magnet was moved, while in Henry's experiment the magnetic flux changed because the current in the first coil was varied. More generally, the magnetic flux through a closed conducting circuit may change whenever the magnitude of the magnetic field, the direction of the magnetic field, or the magnitude of the area enclosed changes. To summarize, whenever there is a change in the magnetic flux through a plane area, there is an electromotive force induced around the boundary of the region.

If we use the subscript 1 to designate quantities at the initial instant and the subscript 2 to designate quantities at an instant a time t later, Eq. 77 takes the form:

$$\mathcal{E} = \frac{(BA)_1 - (BA)_2}{t} \tag{78}$$

We see from Eq. 78 that the unit of electromotive force must be a unit of magnetic field strength multipled by an area unit and divided by a time unit. If we refer to Sec. 4.3 we see that the 1 weber/m^2 = 1 nt/coul-m/sec according to the definition of magnetic field strength given in Eq. 69. Thus, in the system of units used in this book, electromotive force is measured in the units below:

$$\mathcal{E} = \frac{(\text{nt/coul-m/sec})(\text{m}^2)}{\text{sec}} = \text{nt-m/coul} = \text{joules/coul} = \text{volts}$$

As we discussed earlier in this section, an electromotive force is measured in volts and the term is only applicable to a device which is reversible. In the preceding discussion we have shown that a magnetically induced electromotive force is properly measured in volts. This is an example of internal consistency of physical units.

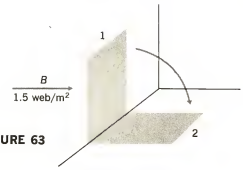


FIGURE 63

As an example of the foregoing, let us consider what happens when a coil consisting of 1000 turns each of area 0.1 m^2 is rotated through 90° in a region in which there is a uniform magnetic field of 1.5 webers/m^2. Let us assume further that the initial orientation of the coil was at right angles to the magnetic field, while the final position of the coil had the plane of the coil parallel to the magnetic field. Fig. 63 illustrates these ideas. Thus, there was no magnetic flux through the coil in the final position, as no part of the magnetic field was at right angles to the plane of the coil. If the time required for this rotation was 0.1 second, the average electromotive force induced in the coil can be calculated from Eq. 78 to have the following value:

$$\mathcal{E} = 1000 \frac{1.5 \times 0.1}{0.1} = 1500 \text{ volts}$$

The factor 1000 appears in the calculation above because there are 1000 turns in the coil and each turn has induced in it the same electromotive force.

We might now point out that our whole society would be different if induced electromotive forces could not be produced. Generators of electrical energy are devices in which large coils are rotated in magnetic fields, so that electromotive forces are induced in the coils. Since energy is conserved in such a process, mechanical energy is transformed into electrical energy, although there are small losses due to electrical resistance and mechanical friction. This electrical energy is then sent for many miles through electrical conductors and converted back into

various useful forms of energy. If this were not possible, the sources of mechanical energy, such as coal, flowing water, and moving air, would have to be transported to the point at which this energy was needed. Ours is truly a society based upon electrical energy.

SUMMARY

Since electrical effects were first noticed for the case of motionless charges, we discussed this topic first. From various experiments we concluded that there are two kinds of electricity, called positive and negative, and that like charges repel one another while unlike charges attract one another. Quantitatively, we found that the force between charges is proportional to the product of their magnitudes and inversely proportional to the square of their distance of separation. We then extended our ideas by defining the electric field at a point as the force on a unit positive charge placed at that point and the potential difference between two points as the work required to move a unit positive charge from one point to the other.

The much more important topic of the effects of moving electrical charges was next considered. The flow of electric charge past a given point per second is defined as the electric current, which is measured in amperes. For the case of many metals it is found experimentally that the current through a conductor and the potential difference between its ends are proportional, which is known as Ohm's law. Energy exchanges when electric currents flow through conductors finished our discussion of this topic.

Historically effects which were called magnetic were known thousands of years ago, although today we now consider all such effects to be caused by moving charges. We discussed the idea of magnetic poles and the magnetic field at a given point. We found that a magnetic field will exert a force on a moving charge and conversely a moving charge will establish a magnetic field near it. This led to the definition of the ampere, which is the unit of electrical current.

This is the fundamental electrical quantity in the system of units used in this book. We next discussed experiments which show that a changing magnetic field will cause electrical charges to move and that the charges will gain energy. This led us to define the term electromotive force as measuring the ability of a device to convert reversibly electrical energy into other forms of energy and vice versa.

PROBLEMS

1 *By what factor is the force between two charges changed if the distance between them is doubled?* ANS.: $\frac{1}{4}$.

2 *If each of two charges is tripled, their separation remaining the same, by what factor is the force between them changed?*

3 *Compute the charge passing a point when a current of 5 amp flows for 2 min.* ANS.: *600 coul.*

4 *Compute the force between a charge of -5 microcoulombs and a charge of -10 microcoulombs separated by a distance of 5 cm. Is the force an attraction or a repulsion?*

5 *Compute the magnitude of a charge which will repel an equal charge with a force of 10 nt when the charges are 0.1 m apart.* ANS.: 3.33×10^{-6} *coul.*

6 *The electric field of the earth is approximately 100 nt/coul. Compute the force exerted by this electric field on a charge of 5×10^{-6} coul.*

7 *Take the radius of the earth as 7000 km and assume that the earth's electric field is caused by a point charge placed at the earth's center. Compute the magnitude of the charge which would be required to produce an electric field of 100 nt/coul at the surface of the earth.* ANS.: 5.44×10^{5} *coul.*

8 *Compute the electric field produced at a distance of 0.2 m from an electric charge of magnitude 2×10^{-6} coul.*

9 *If the work required to move a charge of 3 coul between two points is 60 joules, what is the potential difference between these two points?* ANS.: *20 volts*

10 *How much work is needed to move a charge of 4 coul from one point to another, if the potential difference between the two points is 50 volts?*

11 *A 100-watt electric light bulb uses a current of approximately 1 amp. Compute the charge passing through the light bulb during 20 sec.*

ANS.: *20 coul.*

12 *If a light bulb uses 1 amp of current when it is connected across a potential difference of 115 volts, compute the electrical resistance of the light bulb.*

13 *Refer to Problem 12. How much power does the light bulb use?*

ANS.: *115 watts.*

14 *If the oven of an electric stove uses 3000 watts at a voltage of 220 volts, compute the current used by the oven.*

15 *Refer to Problem 14. If your power company charges 3¢ per KWH, how much would it cost to operate the oven of your stove for half an hour?*

ANS.: *4.5¢*

16 *If the earth is thought of as a very large magnet, is the North Pole of the earth a north or south magnetic pole?*

17 *A charge of 3 coul moves at a speed of 10 m/sec in a direction at right angles to a magnetic field of 0.5 webers/m^2. Compute the force on the charge.*

ANS.: *15 nt.*

18 *Let us assume that the earth's magnetic field is 5×10^{-5} webers/m^2. What current must flow in a conductor 10 m long at right angles to the earth's magnetic field if the conductor is to experience a force of 2 nt?*

19 *A current of 10 amp flows in a conductor 8 cm long which is at right angles to a magnetic field of 1.5 webers/m^2. Compute the magnetic force on the conductor.*

ANS.: *1.2 nt.*

20 *Calculate the magnetic field at a distance of 10 m from a power line carrying a current of 2000 amp. What effect might this current have on a compass used below this power line?*

21 *Find the force per meter between two parallel conductors, each carrying a current of 1000 amp and separated by a distance of 0.05 m.*

ANS.: *80 nt/m.*

22 *If the earth's magnetic field of 5×10^{-5} webers/m^2 is at right angles to a coil of area 2 m^2, compute the magnetic flux through the coil.*

23 *Refer to Problem 22. If the coil is turned 90° to a position where the earth's magnetic field is parallel to the plane of the coil during a time of* $\frac{1}{20}$ *second, compute the electromotive force induced in the coil.*

ANS.: 2×10^{-3} *volt.*

24 *A coil of area 0.2* m^2 *is to be rotated from a position in which the magnetic field is 2 webers/*m^2 *at right angles to the plane of the coil to a second position in which the magnetic field is parallel to the plane of the coil. Compute the time for this rotation if an electromotive force of 5 volts is to be induced in the coil.*

25 *By what factor must the distance between two charges be changed if the force between them is to be tripled?* ANS.: *0.577.*

26 *Suppose that each of two charges is quadrupled, while the distance between them is doubled. By what factor is the force between the charges changed?*

27 *Compute the time a current of 2 amp would have to flow past a given point so that the total charge passing the point is 300 coulombs.*

ANS.: *150 sec.*

28 *If a charge of 600 microcoulombs flows out of a capacitor during a time of* 10^{-3} *sec, compute the average current flowing out of the capacitor during this time.*

29 *Compute the force between a charge of 4 microcoulombs and a charge of* -6 *microcoulombs when they are separated by a distance of 8 cm. Is the force an attraction or repulsion?*

ANS.: 3.37×10^{1} *nt; attraction.*

30 *If two equal charges repel one another with a force of 5 nt when they are 0.2 m apart, what is the magnitude of each charge? Are both charges of the same sign?*

31 *How far away from a given point must a charge of* 6×10^{-6} *coul be to produce an electric field of 3 nt/coul?* ANS.: *134 m.*

32 *At a certain distance from a charge of* 5×10^{-6} *coul the electric field due to this charge is 2 nt/coul. How far is the point in question from the charge?*

33 *If the work required to move a charge of 5 coul from one point to another is 300 joules, what is the potential difference between these points?*

ANS.: *60 volts.*

34 *How much work is required to move a charge of 5 coul between two points if the potential difference between these points is 100 volts.*

35 *The work to move a certain charge between two points between which there is a potential difference of 12 volts is found to be 24 joules. Compute the magnitude of the charge.* ANS.: *2 coul.*

36 *If one burner of an electric stove uses a power of 500 watts, compute the current drawn by this burner at a voltage of 110 volts.*

37 *Refer to Problem 36. Compute the resistance of the burner.*
ANS.: *24.2 ohms.*

38 *Refer to Problem 36. If your power company charges 2 cents per kilowatt-hour, how much would it cost to operate this burner for 3 hours?*

39 *A flashlight draws 10 milliamperes from a 3-volt battery. How much power is used by the flashlight?* ANS.: 3×10^{-2} *watts.*

40 *Compute the speed at which a particle of charge* 1.6×10^{-19} *coul must travel at right angles to a magnetic field of strength* 5×10^{-5} *webers/m*^{2} *so that the force on the particle will be* 10^{-32} *nt.*

41 *A charge of* 3.2×10^{-19} *coul moves with a speed of* 10^7 *m/sec at right angles to a uniform magnetic field of strength 2 webers/m*^{2}*. Compute the force on this charge.* ANS.: 6.4×10^{-12} *nt.*

42 *Refer to Problem 41. If the mass of the particle is* 6.8×10^{-27} *kg, compute the acceleration of the particle.*

43 *Compute the magnetic field strength such that a particle of charge* 1.6×10^{-19} *coul will experience a force of* 10^{-12} *nt when it moves with a speed of* 10^9 *cm/sec in a direction at right angles to the magnetic field.*
ANS.: 6.25×10^{-1} *webers/m*^{2}*.*

44 *Compute the magnitude of the current flowing in a conductor 5 m long and directed at right angles to the earth's magnetic field of* 6×10^{-6} *webers/m*^{2} *so that the conductor will experience a force of 1.5 nt.*

45 *A conductor of length 2 m carries a current of 5 amperes at right angles to a uniform magnetic field of 0.5 webers/m*^{2}*. Compute the force on this conductor.* ANS.: *5 nt.*

46 *A conductor of mass per unit length 5 gm/m is at right angles to a uniform magnetic field of strength 1.5 webers/m*^{2}*. Compute the current*

which this conductor must carry so that the magnetic force will equal its weight.

47 *A conductor carries a current of 100 amp. How far from this conductor will the magnetic field of the conductor be no more than 10^{-4} webers/ m^2?* ANS.: *0.2 m.*

48 *How far apart must two parallel conductors, each carrying a current of 2000 amp, be for the force per unit length between them to be less than 0.5 nt/m?*

49 *A square coil of 50 turns, each 10 cm on a side, is at right angles to the earth's magnetic field of 5×10^{-5} webers/m^2. What is the total flux through the coil?* ANS.: *2.5×10^{-5} webers.*

50 *Refer to Problem 49. If the coil is rotated 90 degrees during 0.10 sec, what is the emf induced in the coil?*

51 *Refer to Problems 49 and 50. What is the average current in the coil while it is being rotated if its resistance is 5 ohms?*
ANS.: *5×10^{-5} amp.*

DISCUSSION QUESTIONS

1 *Electrostatic experiments usually work poorly on days when the humidity is high. Why is this?*

2 *When you comb your hair in dry weather the comb will attract small pieces of paper or lint, even though they are not charged. Explain how this can happen.*

3 *Is it possible to arrange two electrical charges so that at least at one point the net electrical field they produce is zero? If so, sketch the arrangement and state any other necessary conditions for this to be so.*

4 *Prove that the electrical field inside a perfect conductor (a material in which the charges are completely free to move) is zero.*

5 *Prove that the electrical field at the surface of a perfect conductor must be at right angles to the surface of the conductor.*

6 *Justify the statement that if any charge is put on a perfect conductor, it will reside entirely on the surface of the conductor.*

7 *The electron is negatively charged. Will it move in the direction of the electrical field or in the opposite direction? Will the electron move towards regions of high or low potential?*

8 *Suppose that the potential difference is zero between all points in a certain region. Can you say anything about the electrical field in this region?*

9 *Consider two electric light bulbs rated at 60 watts and 100 watts respectively when operated on 115 volts. Which has the higher resistance?*

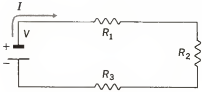


FIGURE 64

10 *Resistors are said to be connected in* series *when they are connected as shown in Fig. 64. In this case, exactly the same current flows through each resistor.*
Show that the current delivered by the battery is given by:

$$I = \frac{V}{R_1 + R_2 + R_3}$$

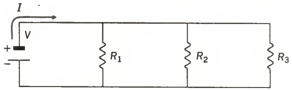
As far as the battery is concerned, it is delivering current to an equivalent resistance, *R, given by:*

$$R = R_1 + R_2 + R_3$$

11 *When resistors are connected so that their end-points are joined by perfect conductors, we say that they are connected in* parallel. *In this situation, exactly the same potential difference exists across each resistor. The circuit is shown in Fig. 65. Show that the current through each resistor is given by one of the expressions:*

$$V = I_1R_1 \qquad V = I_2R_2 \qquad V = I_3R_3$$

FIGURE 65



From the equations above show that the total current delivered by the battery is:

$$I = I_1 + I_2 + I_3$$
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Thus, as far as the battery is concerned show it is delivering current to an equivalent resistance R *given by:*

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

12 *On a dry day it is possible to generate a potential difference greater than ten thousand volts merely by walking along a rug. Explain why this is not dangerous, while an ordinary household electrical outlet furnishing a potential difference of 115 volts may be lethal.*

13 *Explain the difference between the terms "potential difference" and "electromotive force." Is it possible to have one without the other?*

14 *A positively charged particle moves through a region without being deflected from its straight line path. Can we say with confidence that in this region the electric field or the magnetic field is zero, or both are zero?*

15 *A charged particle enters a region and is deflected sideways. What can we say about the magnetic and electric fields in this region?*

16 *Circular rings made of copper and wood are placed in the same varying magnetic field. Are the electromotive forces induced in the rings the same? Are the currents induced in the rings the same?*

17 *Two iron bars attract one another, no matter which ends are brought near. Do you conclude that both are magnetized? If you conclude that only one is a magnet, how would you tell which one was the magnet?*

CHAPTER FIVE

WAVE MOTIONS

5.1 General properties of waves

Nearly everyone has sat on the seashore and watched waves roll in. It is less evident that the sound our ears hear and the light our eyes see are also carried by waves. In this section we will discuss those properties of waves which are essentially the same for all types of wave motions. Later in this chapter we will treat particular types of wave motions.

Even the most casual observation of water waves shows that these waves carry *energy,* which is often quite large. This is a general characteristic of all waves, and may be measured in

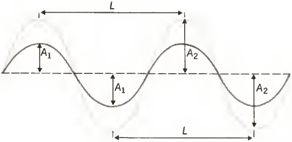


FIGURE 66

terms of the energy transferred per second. In the metric units used in this book, the rate at which a wave delivers energy from a source to a receiver would be measured in joules/second or watts. Here it should be noted that the material carrying the disturbance (water in the case here considered) merely acts as a device for transferring energy from one point to another, without the material being permanently changed. Similarly, the noise and energy of a thunderclap are transmitted to our ears by the intervening air, without the air itself being much affected. In a less obvious way, energy is transferred from a radio transmitter to our receiver by means of radio waves.

If we watch water waves first on a calm day and then on a day on which the water is quite rough, we would decide that the higher the waves, the more energy they brought in to shore. Suppose we measure the disturbance of the water compared to its average or completely still level. The height of the crest of a wave above the average level of the water or the depth of the trough of a wave below the average water level is called the *amplitude* of the wave. Waves of two different amplitudes are shown in Fig. 66. For simple waves it is found that the energy carried by the wave per unit of time is proportional to the square of the amplitude of the wave. Thus, doubling the amplitude increases the energy carried by the wave by a factor of four.

Another simple observation we can make on water waves is the speed at which a given crest or trough travels past a fixed point. We could easily do this by standing on a dock or an anchored boat. This speed is known as the *wave velocity* v and is measured in meters per second. As examples, water waves

typically have velocities of a few meters per second, sound travels in air at a velocity of about 330 meters per second, and the velocity of a light wave is about 300 million meters per second. The velocity of a given type of wave depends primarily upon the material which transmits the wave, but it may also depend on other factors, such as the temperature.

If we observe water waves from a fixed point, such as a dock, we can easily measure the number of crests or troughs which pass us per second. This quantity is known as the *frequency* f of the wave. Frequency is variously referred to in terms of waves per second, vibrations per second, and cycles per second, which is often abbreviated as *cps*. All three of these terms mean the same thing, namely, the number of waves passing a given point per second. In the same situation, we could also measure the time interval between two successive crests or troughs. This time is known as the *period* T of the wave and is measured in seconds. Clearly, if f waves pass a given point per second, then the time interval between successive waves is $1/f$ seconds. Since this time interval has been defined to be the period T of these waves, we have the useful relation:

$$T = \frac{1}{f} \tag{79}$$

As an example of the use of Eq. 79, suppose that water waves strike a beach every 10 seconds. Then the period of these waves is 10 seconds and their frequency is $f = \frac{1}{10}$ per second. A sound wave might transmit a sound wave of frequency of 250 vibrations per second, which is just below the frequency of middle C. The period of such a wave is then given by $T = \frac{1}{250} = 4 \times 10^{-3}$ second. As a last example, a frequency in the FM radio band is 100 megacycles or 10^8 cycles per second. The period of these waves is then given by $T = 1/10^8 = 10^{-8}$ second.

If we observe water waves from the edge of a dock and have a friend to help us make measurements, we can measure the distance between two successive crests or between two successive troughs, as is shown in Fig. 66. This distance is called the *wavelength* L of these waves and is measured in meters.

Wavelengths of water waves may range from a very small fraction of a meter for ripples up to many meters for ocean waves.

Suppose that we observe that f waves each of wavelength L pass a given point during one second. Then the distance covered by the first crest during this second will be exactly fL. However, the distance covered by any given crest (including the first) during one second was earlier defined as the velocity of the wave. In terms of an equation, we then have the following relation between the frequency f of a certain wave, its wavelength L, and its velocity v:

$$v = fL \tag{80}$$

We can replace the frequency f in Eq. 80 by the period T if we use Eq. 79. We find then:

$$v = \frac{L}{T} \tag{81}$$

Let us compute the wavelength of a sound wave of frequency 300 vibrations per second travelling at a speed of 330 meters per second. According to Eq. 80, this wavelength is $L = \frac{330}{300} =$ 1.1 meters. If the distance between successive crests of a water wave is 20 meters and if waves strike the beach every 5 seconds, then the speed of these waves is given by $v = \frac{20}{5} = 4$ m/sec. Finally, if radio waves travel with a speed of 3×10^8 m/sec, the frequency of waves which have a wavelength of 1 centimeter is given by $f = 3 \times 10^8/10^{-2} = 3 \times 10^{10}$ cycles per second $=$ 30 gigacycles. From the preceding examples it should be clear that values of velocity, frequency, wavelength, and period cover an enormous range.

If we watch a cork floating on water as waves pass, we see the cork bob up and down as the waves move horizontally. The

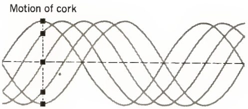


FIGURE 67

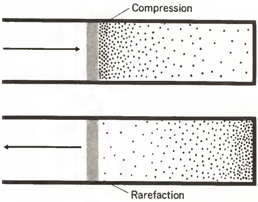


FIGURE 68

situation is shown in Fig. 67. In this case the vibrations of the material are at right angles to the direction of motion of the wave itself, so we call this a *transverse wave.* Similarly, the waves on a stretched wire, such as a violin string, travel along the length of wire, while the parts of the wire move at right angles to the wire's length. Thus, waves on stretched wires are also transverse waves. A less obvious example of a transverse wave is a light or radio wave, in which variations in electric and magnetic fields take place in a direction at right angles to the direction in which the disturbance itself moves.

Let us next consider moving a piston suddenly along the length of a tube, as shown in Fig. 68. The rapid motion of the piston compresses the air just ahead of the piston, so that a pulse of high pressure travels ahead of the piston. On the other hand, if the piston is suddenly moved in the opposite direction, the pressure near the face of the piston is lower than normal air pressure, so that a region of low pressure travels down the tube. If a drumhead vibrates back and forth, alternating regions of high and low pressure are transmitted to the air. When these variations in pressure reach our ears, we experience the sensation of sound. In this example the direction of the motion of the drumhead and the direction in which the sound wave moves are parallel, so we call this type of wave a *longitudinal wave.* If you hammer on one end of a metal rod, this impulse will be transmitted to the other end of the rod as a longitudinal wave.

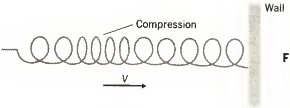


FIGURE 69

Similarly, if you take a spring and attach one end to a wall while you hold the other end in your hand, you can easily see a longitudinal wave. After the spring has been stretched a bit, suddenly move your hand towards the wall. A compression will then travel down the spring, as is shown in Fig. 69. This compression is an example of a longitudinal wave.

We will now try to make clear the difference between transverse and longitudinal waves. In the case of a transverse wave the particles of the material move back and forth at right angles to the direction in which the wave and the energy carried by it travel. The waves we see on water provide an example of such a wave. If the particles of the material move back and forth along the same direction as the wave travels, we have a longitudinal wave. The compression described in the preceding paragraph and shown in Fig. 69 is an example of a longitudinal wave. In a solid it is possible for both types of waves to be transmitted. For instance, when an earthquake shakes a part of the earth, both longitudinal and transverse waves travel away from the location of the earthquake.

Here we should point out that the general properties of waves are the same, regardless of whether they are transverse or longitudinal. All waves transmit energy from a source to a receiver without appreciably affecting the transmitting material. The amplitude of any simple wave is always the maximum departure of some property of the material from its average or undisturbed value. For instance, the amplitude of a water wave is the height of a crest or the depth of a trough as compared to the level of the undisturbed water. In a sound wave the amplitude is the maximum amount that the air pressure differs from the pressure of still air. Similarly, the velocity of any wave is the

speed at which some characteristic of the wave travels through the material, and the frequency of any wave is the number of complete waves or vibrations which pass a given point in one second. Finally, the wavelength is the distance between two successive points at which the disturbance caused by the wave in the material is the same. From the above discussion we see that the general properties of all waves are the same, regardless of the type of wave or the material in which it is transmitted.

5.2 Sound waves

One of the commonest waves in everyday life is the sound wave. Some vibrating object causes changes in the pressure of the air which then travel to our ears or to a microphone. In the case of the ear small bones are affected and nerve impulses sent to our brain are interpreted as sound. When pressure variations strike a microphone, electrical impulses are produced which may be used in various ways. In this section, however, we will be more concerned with sources of sound than with receivers.

Suppose that we disturb some point on a stretched wire, as we could by bowing a violin string. This disturbance will have a frequency of f vibrations per second and a wavelength of L meters. Thus, a disturbance will travel towards the ends of the wire with a velocity v given by $v = fL$. We will assume that the ends of the wire are fixed, so that they cannot absorb energy from the wave. Therefore, the energy carried by the wave will be reflected as shown in Fig. 70. As the diagram shows, the displacement of the reflected wave at the fixed end of the wire is equal and opposite to the displacement of the incoming wave.

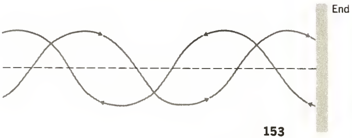


FIGURE 70

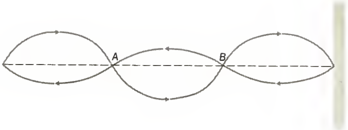


FIGURE 71

Thus, at the end of the wire the sum of the incoming and reflected waves is zero. This is exactly what we would expect, since the end of the wire is not free to move. At other points on the wire the displacement is the sum of the displacements of the incoming and reflected waves. In general, the displacement at any point along wire is not a constant, but varies with time.

We will now consider a complete wire with both ends fixed. Waves will travel along the wire in both directions and will be reflected at the ends as described in the preceding paragraph. If the length of the wire and the wavelength of the waves are properly related, we can now have *standing waves* on the wire. All that is required is that the incoming and reflected waves at each end of the wire have zero displacement at all times. This is shown in Fig. 71. This will insure that there will be no displacement at either end of the wire. Furthermore, points such as A and B in the diagram will also have no displacement. Points such as A and B are known as *nodes.* The left end of the wire could be located at any node, since at such a point it would have no displacement. In general, we will have a standing wave whenever both ends of the wire are located at points where the incoming and reflected waves cause zero displacement.

Since a simple wave has two zeroes of displacement in each complete wavelength, as long as any zero reaches an end of the wire exactly we will have a standing wave. From this we see that we will have a standing wave if the length of the wire is some integral (whole) number of half-wavelengths. If the length of the wire is L' and the wavelength of the standing wave is L, we must satisfy the following relation in order to have a standing wave:

$$L' = n\frac{L}{2} \qquad n = 1, 2, 3, \ldots \tag{82}$$

From Eq. 82 we see that for a wire of fixed length the wavelengths which produce standing waves (and thus waves of long duration) are given by:

$$L = \frac{2L'}{n} \tag{83}$$

If we refer to Eq. 80 we find that for a given wire the possible frequencies which produce standing waves are given by the equation:

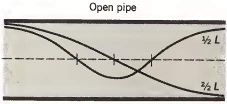
$$f = \frac{v}{L} = \frac{nv}{2L'} \tag{84}$$

Since the lowest frequency, f_1, which can produce a standing wave is given by $n = 1$, $f_1 = v/2L'$. Thus, the frequencies of a stretched wire are all related to the lowest frequency by the equation:

$$f = nf_1 \qquad n = 1, 2, 3, 4, \ldots \tag{85}$$

The lowest frequency of a stretched wire is called its *fundamental frequency*. Its higher frequencies are called *harmonics* by scientists and *overtones* by musicians. If $n = 2$, the frequency is $2f_1$, which is called either the second harmonic or the first overtone. Similarly, if $n = 3$, we obtain the third harmonic or the second overtone. From the above we see that in the case of stringed instruments, such as the violin, guitar, and piano, all harmonics are possible.

In the case of the wind instruments, such as the trumpet, flute, and organ pipes open at both ends, again we find that all harmonics occur, so that n may take any integral value in Eq. 85. On the other hand, only odd values of n are possible for a pipe closed at one end, such as a jug or certain organ pipes. Examples are shown in Fig. 72. Thus, a jug or an organ pipe closed at one



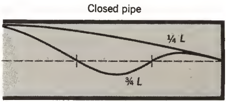


FIGURE 72

end has a different sound, since all of the even harmonics are missing. Finally, instruments such as the drum and the cymbal produce harmonics, but these harmonics are not related to the fundamental frequency by an integer. These percussion instruments do not give a sound anything like the other instruments and are mainly used for rhythmic emphasis.

The note C three octaves below middle C has a frequency of about 32 vibrations per second, while the C three octaves above middle C has a frequency of about 2048 vibrations per second. (For this example, we have taken middle C to have a frequency of 256 vibrations per second, while in practice musicians use a frequency of 260–264 vibrations per second.) At this point the reader might ask why sound reproduction equipment is commonly designed to produce frequencies up to 5000–20,000 vibrations per second. The reason is that a musical instrument does not sound realistic unless the harmonics it produces are also heard. Since the normal human ear is sensitive to frequencies above 10,000 vibrations per second, it is necessary to reproduce quite high frequencies if a musical sound at a fundamental frequency of 1000 vibrations per second is to sound natural. If the relative energies present among the harmonics is changed by the reproducing apparatus, the sound is unreal and is said to be distorted. One central problem in the reproduction of music is to present the original harmonics of each instrument in their proper relationship to one another.

5.3 Light waves

In addition to sound waves, we experience light waves constantly in life. Where sound waves are longitudinal, light waves are transverse. Another difference is that the velocity of sound waves in air is about 330 m/sec, while the velocity of light waves in empty space is 300 million m/sec. Thus, we see a lightning flash almost instantaneously, but seconds pass before we hear the accompanying clap of thunder. There are other differences

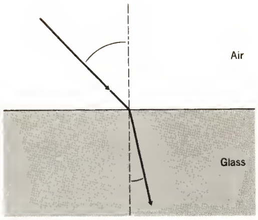


FIGURE 73

between these two types of waves, although both show the basic properties of waves described in Sec. 5.1.

In this section we will present one piece of evidence which shows that light is a wave. In Sec. 5.4 and 5.5 we will consider additional experiments which prove the same statement. It should be noted that historically the experiments described in this section were performed later than the ones referred to in the following two sections.

Suppose that a beam of light travels from air into a transparent material, such as glass or water. It is found that the light ray's direction is changed when it enters the material. This is known as *refraction* and is shown in Fig. 73. Following custom, the direction of the light ray is indicated by measuring its angle with respect to the normal or perpendicular line to the interface. In the example shown in the diagram the light ray is bent closer to the normal as it passes from air into the glass.

In the seventeenth century and for many years later, two theories of the nature of light were proposed. According to one theory, light consists of a stream of particles. The other theory said that light was a wave. We will next discuss how each theory explains refraction and how measurements of the speed of light in air and water decided the question in favor of the wave theory of light.

Let us first treat light as a stream of particles. Experimentally we find that light rays are reversible, in the sense that they follow the same path whether going from air into a material or going from the material into air. If we resolve the velocity of a particle of light into components parallel and perpendicular to the interface between air and the material, by symmetry the velocity component parallel to the interface will not change as the particle travels from air into the material. In order to explain the bending of the light ray closer to the normal in the material, we must assume that some sort of force accelerates the particle as it passes from air into the material. Thus, the component of the particle's velocity parallel to the normal must be greater in the material than in air. These relations are shown in Fig. 74. According to this theory of the nature of light, the velocity of light should be greater in the material than it is in air.

Let us now consider refraction on a wave theory of light. If we consider a wave front, *AB*, travelling at a speed v in air, the only way in which the ray can have its direction changed to be closer to the normal in the transparent material is for the wave to travel at a smaller speed in the material. These relations are

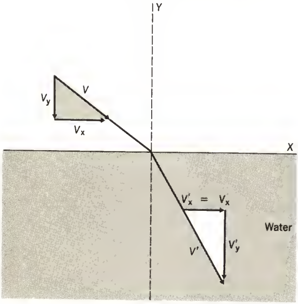


FIGURE 74

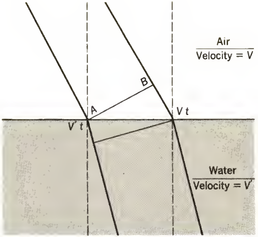


FIGURE 75

shown in Fig. 75. Thus, we see that to explain refraction on a wave theory of light, the velocity of light in a material must be less than in air (vacuum, to be precise).

The considerations of the two preceding paragraphs allow us to decide whether light consists of particles or waves. If the velocity of light is greater in water, for instance, than it is in air, light consists of particles. Conversely, if the velocity of light in water is less than it is in air, light consists of waves. Experiments performed by Foucault in 1850 conclusively showed that the velocity of light in water is less than it is in air, so that light must be a wave motion.

The velocity of light in vacuum is conventionally given the symbol c and has a value of approximately 3×10^8 m/sec. Let us call the velocity of light in a material medium v. Then the *index of refraction* of the material, n, is defined by the equation:

$$n = \frac{c}{v} \tag{86}$$

Some typical values of n for some common substances are given in the table on the next page. The reader will note that for all except the most precise measurements, air can be treated as a vacuum as regards its index of refraction. From the discussion

Material	n	Material	n
Dry air	1.0003	Glycerine	1.470
Water	1.333	Benzene	1.501
Ethyl alcohol	1.354	Crown glass	1.517
Acetone	1.359	Flint glass	1.627
Chloroform	1.446	Carbon disulphide	1.628
Quartz	1.458	Diamond	2.419

INDEX OF REFRACTION FOR COMMON MATERIALS

of refraction above on the wave theory of light, it is evident that a light ray is deflected more when entering a material of high index of refraction than it would be when entering a material of low index of refraction.

In general the index of refraction of a material varies slightly with the wavelength of the light used. This is known as *dispersion.* If a beam of light is passed through a prism, as is shown in Fig. 76, different wavelengths will be refracted through different angles. It happens that for glass the long wavelengths (red) are refracted least and the short wavelengths (violet) are refracted most, as is shown in the diagram. The array of colors which emerges from the prism is known as a *spectrum.* Probably the commonest example of a spectrum is the rainbow, which is caused by the dispersion of light by water droplets. From observations of this sort, we conclude that white light (sunlight) is a mixture of colors (wavelengths). In the next chapter we will discuss the origin of these various wavelengths.

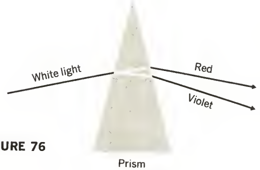


FIGURE 76

Most of the energy in the sunlight reaching our eyes after passing through the atmosphere lies in the region of wavelengths between about 4×10^{-7} meter and 7×10^{-7} meter. It is therefore not surprising that through the process of natural selection the human eye responds chiefly to wavelengths in this range, which is known as the *visible spectrum.* The shortest wavelengths correspond to the color violet, and then as we look at longer and longer wavelengths we get the sensation of blue, green, yellow, orange, and red. A chart showing the relation between approximate wavelengths and corresponding colors is shown in Fig. 77. Here it should be pointed out that certain colors, such as brown, do not correspond to a single wavelength. The sensations we name by these colors are produced by mixtures of wavelengths. Producing various colors by mixing is used in color photographs, color television, and color printing.

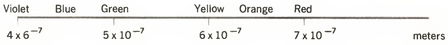


FIGURE 77

Suppose we take a wavelength of 6×10^{-7} meter (yellow) as a typical wavelength of visible light. Since the velocity of light in empty space (air, essentially) is 3×10^8 m/sec, the frequency of such a wave is given by $f = (3 \times 10^8)/(6 \times 10^{-7}) = 0.5 \times 10^{15} = 5 \times 10^{14}$ cycles per second. To show how large this frequency is, we might note that a typical standard radio frequency is about 10^6 cycles per second while a typical television broadcasting frequency is about 10^8 cycles per second. From this it is easy to see that showing that light is a wave is difficult indeed, since the interval between the impact of successive waves on the eye is so short.

In the case of sound waves, the disturbance which is propagated as a wave is a series of variations of pressure above and below the average pressure in the material. Light waves consist of a propagation through space of variations in electric and magnetic fields. Thus, light waves are called *electromagnetic*

waves. The electric and magnetic fields occur in directions at right angles to the direction in which the effect travels, so these are transverse waves. The reader should contrast these waves with sound waves, in which the particles of the transmitting material (air, for example) move in a direction parallel to the direction in which the disturbance moves. This is one important difference between light and sound waves.

The thoughtful reader might guess at this point that there might exist electromagnetic waves of frequencies or wavelengths to which the human eye is not sensitive. This question was answered about a century ago by Maxwell, who predicted that there should be electromagnetic waves of all frequencies and wavelengths. Furthermore, all of these waves should travel at the speed of light, namely, 300 million m/sec. At the time the theory was proposed by Maxwell, the only electromagnetic waves known were light waves. While the theory did account for the properties of light waves very well, we expect a theory to do more than merely explain known facts. In 1887 a German physicist, H. R. Hertz, succeeded in producing electromagnetic waves with frequencies greatly different from optical frequencies. Since these waves also obeyed Maxwell's theory, the electromagnetic wave theory was accepted.

During this century electromagnetic waves of a wide range of frequencies have been observed and used for various purposes. Frequencies in the range 10^4–10^9 cycles per second are used for radio and television communications. Frequencies of approximately 10^{12}–10^{14} cycles per second lie in the *infrared* range, which is often used for heating purposes. Frequencies in the range roughly of 10^{15}–10^{17} cycles per second lie in the ultraviolet region, while frequencies greater than about 10^{18} cycles per second are known as *x rays.* A chart showing some of the commonly used frequencies is shown in Fig. 78.

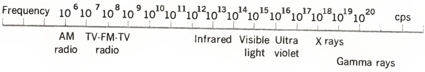


FIGURE 78

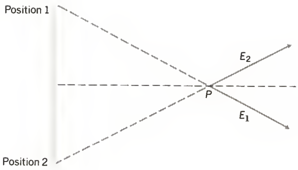


FIGURE 79

We will now explain qualitatively how the variations in electric and magnetic fields which we call electromagnetic waves are produced for frequencies used in radio and television. (A different mechanism is needed to explain electromagnetic waves of much greater frequency.) Suppose that an electric charge (an electron, usually) is made to move back and forth in a regular manner along a metal rod (antenna), as shown in Fig. 79. The electric field produced by the charge at a point such as P will therefore change as the charge moves along the length of the rod. In addition, a moving charge is equivalent to an electric current, so that a changing magnetic field is also produced at the point P. The triumph of Maxwell's theory was in predicting that these variations of electric and magnetic fields would be propagated through space with a constant velocity equal to the known velocity of light. Furthermore, the velocity did not depend on the particular frequency at which the charge oscillated back and forth along the metal rod.

A simple radio antenna consists of a straight metal rod in which there are many electrons free to move. When a varying electric field from a distant transmitting antenna reaches the receiving antenna, the electrons experience forces and move back and forth along the length of the receiving antenna. This oscillatory motion of the electrons is the same as an oscillatory current. This current is amplified by the radio and eventually is converted into sound waves by the loudspeaker. Again, it should be noted that the mechanism of the reception of electromagnetic waves of much higher frequency is neither as well

understood nor as simple as is the case for radio waves. Nevertheless, in all cases the source emits electromagnetic waves because of the motions of charged particles and the detector responds to the energy carried by these waves in one fashion or another.

Information may be transmitted by radio waves if some property of the waves varies in a way proportional to the information. For instance, a variation in the amplitude of the wave or the frequency of the wave can be used. In either case we say that the wave is *modulated,* so that we speak of *amplitude modulation* (AM) or *frequency modulation* (FM). The most common example of AM is the broadcast band in the vicinity of 1 megacycle, while FM is broadcast at a frequency near 100 megacycles. Television combines both forms of modulation by using AM for the picture and FM for the sound.

5.4 Interference of waves

While waves on water or a stretched string can be seen, it is not directly evident that sound and light are effects transmitted by waves. At one time, as mentioned earlier, it was believed that light consisted of a stream of particles. In this section we will discuss the phenomenon of interference, which provides a crucial test of the question of whether a given effect is caused by waves or particles.

Suppose that particles from two sources strike the same point. The effect of two particles hitting the same point will be greater than if only a single particle struck that point. Similarly, if crests of two waves both reach a given point at the same time, their combined effect will be greater than if only a single wave had arrived. The addition at a point of two waves of the same frequency, wavelength, and velocity is shown for various times in Fig. 80. In this example the crests of both arrive simultaneously, as do the troughs and other corresponding points of the two waves. When corresponding points of two or more waves arrive at the same time, we say that the waves arrive *in phase.*

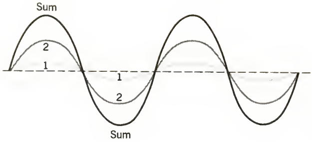


FIGURE 80

The sum of the two waves is then greater than either wave separately. We call this situation *constructive interference.* Here the reader should note that either particles or waves which are in phase can produce a constructive or additive effect at a given point.

When two waves strike a point, it may happen that their crests do not arrive at exactly the same instant. The waves are then said to be *out of phase.* In the extreme case the crest of one wave occurs half a period later than the crest of the other wave, so that the crest of one wave falls on the trough of the other. This situation is shown in Fig. 81. In this case the waves subtract from one another so that the resultant effect is less than would be caused by either wave alone. Here we have an example of *destructive interference.* This partial cancellation of one wave by the other can only happen continually if the waves have the same velocity, frequency, and wavelength, and also have a constant phase relation. One way of achieving a constant phase relation is to have parts of a given wave travel from the source to the detector by different, but constant, paths. Since the times needed for the two waves will be different, one wave will arrive

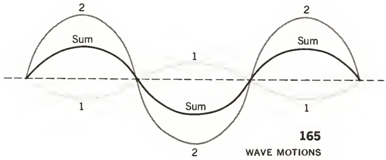


FIGURE 81

later than the other. If the situation is arranged properly, a crest of one wave can be made to fall on the trough of the other, so that destructive interference results. If, in addition, the amplitudes of the two waves are the same, then complete cancellation can be achieved. When complete cancellation occurs, no effect is noticed by the detector.

When two particles strike a detector, the effect is always greater than if only one particle had been used. Thus, cancellation never occurs when particles are involved. As we saw in the preceding paragraph, partial or complete cancellation can occur with waves. Whenever we observe that two effects cancel one another, if only partially, we know that the phenomenon is caused by a wave. On the other hand, if two effects striking a detector produce a greater result than one alone would, we cannot tell if we are dealing with a stream of particles or a wave. Thus, the crucial distinction between an effect caused by particles and one caused by waves is that only a wave can produce destructive interference or cancellation. This is the test on which we base our statement that sound and light are produced by waves.

In order to show that sound is caused by waves, let us consider the common situation in which two loudspeakers are driven by the same amplifier. If the input to the amplifier is a single frequency (from a tuning fork, perhaps), we can find places in front of the loudspeakers at which the sound intensity will be a maximum and other places at which the sound intensity will be nearly zero. The situation is shown in Fig. 82. If wave A has to travel a distance of one wavelength L farther than wave B in order to reach point 1, the crests of wave A will be delayed by exactly one period T compared to the crests of wave B. Thus, the next later crest of wave B will fall on a crest of wave A and constructive interference will occur. At a point such as 2, it may happen that wave A only has to travel a distance of $\frac{1}{2}L$ greater than the distance travelled by wave B, so that wave A lags behind wave B by $\frac{1}{2}T$. In this case a crest of wave B falls on a trough of wave A and destructive interference occurs. Experiments of this sort show that destructive interference

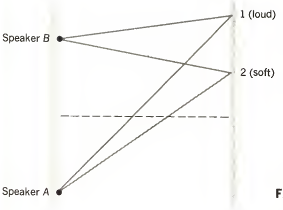


FIGURE 82

can be produced with sound and therefore sound is a wave phenomenon.

In about the year 1800, England's Thomas Young showed that light was caused by waves. His experiment was very similar to that discussed in the preceding paragraph in connection with waves. A narrow beam of light struck two slits and the result was observed on a screen, as shown in Fig. 83. At the point marked 1, the wave from the lower slit had travelled a distance of one wavelength farther than the wave from the upper slit. Thus, at this point crests still fall on crests and troughs on troughs, so we observe brightness at point 1. At point 2 the wave from the lower slit travels a distance only half a wavelength greater than that travelled by the wave from the upper slit. In

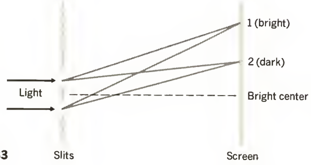


FIGURE 83

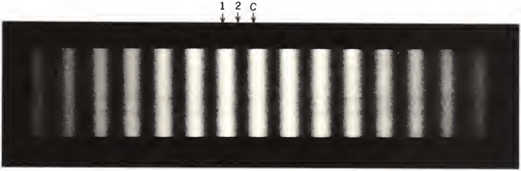


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FIGURE 84

this case a crest of one wave falls on a trough of the other wave or vice versa, so here we observe darkness. A photograph of this effect is shown in Fig. 84, in which the center of the pattern and points 1 and 2 are marked. Since destructive interference is observed at point 2, we conclude that light is a wave motion.

From the geometries shown in Figs. 82 and 83 we can calculate the wavelength involved. For instance, in Fig. 82 we see that the distance $A1$ is related to the distance $B1$ by the equation $A1 - B1 = L$. Since these distances are easily measured, the wavelength L can be computed. Most methods of determining wavelengths use a method based on an interference effect and a knowledge of the distances involved. Thus, interference experiments are used both to show that a given effect is caused by a wave and also to determine the wavelength of the wave. If either the velocity or the frequency of the wave can be found, we then know most of the important properties of the disturbance.

5.5 Polarization of waves

Earlier in this chapter we stated that sound waves are longitudinal, with the vibrations taking place in the same direction as the wave itself moves. We also said that light waves are transverse, with the variations in electric and magnetic fields occurring at right angles to the direction in which the disturbance moves.

In this section we will discuss polarization of waves. As we will see, only transverse waves can be polarized, which allows us to discriminate between the two types of wave motion.

Suppose that we consider a fence with water waves striking it, as is shown in Fig. 85. The up-and-down motion of the water will be greatly reduced if the rails are horizontal obstacles, and only a fraction of the energy carried by the wave will pass through the fence. On the other hand, if the fence has vertical rails as shown in Fig. 86, the vertical motion would be affected only by a small amount, and most of the energy of the wave would get past the fence. In this case we see that the waves are transmitted if there are directions in the obstacle not interfering with the motion of the water. If there are no directions of easy transmission, then the waves will be almost completely stopped. We say then that water waves are *polarized* in a vertical direction, since they can pass by vertical obstacles but not horizontal obstacles.

Now let sound waves strike a set of parallel obstacles similar to those shown in Figs. 85 and 86. Here the vibrations of the material (air) are along the direction of the motion of the wave

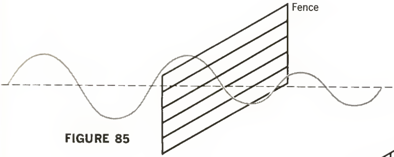


FIGURE 85

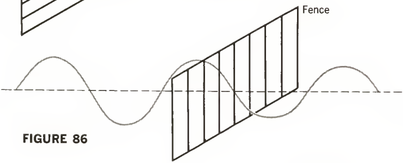


FIGURE 86

itself. If we change the directions of the obstacles, the wave is not affected, since it finds as many chances to pass between the obstacles in one orientation as in any other. Thus, a longitudinal wave is transmitted the same through a set of parallel obstacles, regardless of how the obstacles are rotated. A longitudinal wave does not show any polarization effects. In the case of sound, we find experimentally that there is no polarization effect, so we conclude that sound is a longitudinal wave motion.

Let us now pluck a violin string in a vertical direction. A piece of cardboard with its edge vertical will produce no effect on the string until it is very close to the string. If the edge of the cardboard is now turned until it is horizontal, we notice that the string is hitting the cardboard before its edge is close to the visible position of the string. From this we become convinced that the string is vibrating vertically but not horizontally, so that the wave existing on the string is transverse. With suitable equipment, the amplitude of waves on a string can be made large enough for the eye to see the direction in which the vibration takes place, which is the most convincing evidence that this type of wave is indeed transverse.

In a solid there may be both transverse and longitudinal waves. For instance, earthquake waves in the earth consist of both types. Since the two types of waves travel at different speeds, they take different times to travel from the center of an earthquake to the detecting equipment. Since the speeds of the two waves are known, a measurement of the time difference allows the calculation of the distance the waves have covered. If several observing stations determine these distances, the location of the center of the earthquake can easily be found. From such observations we can also obtain considerable information about the interior structure of the earth.

We can perform a simple experiment to find out if an electromagnetic wave is transverse or longitudinal. If we use wavelengths of the order of centimeters, it is easy to construct a set of parallel, conducting wires similar to the fences shown in Figs. 85 and 86. The oscillating electric field can only pass through such a grid when it is parallel to wires, or what is the

same thing, parallel to the spaces between the wires. If the electric field is at right angles to the wires, the energy of the field is used up in heating the wires. Waves from a straight antenna have their electric fields parallel to the back-and-forth motions of the electrons along the length of the antenna. We observe transmission of the electric field when the grid is parallel to the antenna. After the grid has been rotated through 90 degrees, very little energy is found to pass through the grid. From these observations we decide that radio (electromagnetic) waves are transverse.

While light is an electromagnetic wave, at any instant we observe waves from a large number of sources, which are atoms or molecules. We will discuss this topic in the following chapter. Since these sources are oriented at random, we must first find out how to sort out those waves which have their electric fields all in the same direction. In essence this is done with a device similar to a set of parallel, conducting wires. For instance, a material made by the Polaroid Corporation and called Polaroid consists of a set of parallel crystals of hydroquinone. When light passes through a piece of Polaroid, only those waves or parts of waves which have their electric fields parallel to the crystals are transmitted. The waves emerging from the piece of Polaroid are

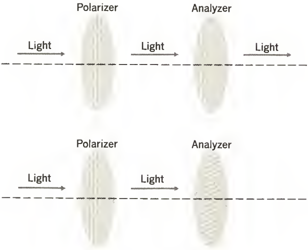


FIGURE 87

then said to be *polarized* parallel to the crystals in the material. To test that these waves are polarized, we use a second piece of Polaroid as an analyzer. When the crystals of the analyzer are parallel to the crystals of the first Polaroid sheet, we observe transmission of light through the analyzing sheet of Polaroid. If we now rotate the analyzer 90 degrees, no light passes through it. These experiments are shown in Fig. 87. Since in these experiments light shows the property of polarization, we conclude that it is a transverse wave. In fact, all electromagnetic waves are transverse, although greatly different devices are needed to show this for the wide range of wavelengths and frequencies known for this type of wave.

*5.6 Mirrors and lenses

In this section we will assume that the objects involved have dimensions which are large compared to the wavelengths of visible light of about 6×10^{-7} meters. This is known as *geometrical optics.* We can ignore the actual wave nature of light and treat light rays as straight lines, since diffraction and interference phenomena are unimportant.

The law of reflection of a ray of light from a polished, plane surface was known by the Greeks. As is shown in Fig. 88, it is customary to measure the angle of incidence i and the angle of reflection r with respect to the normal to the reflecting surface. We can then state the laws of reflection from a plane surface in the following form:

1. $i = r$

2. The incident ray, the normal, and the reflected ray all lie in the same plane.

FIGURE 88

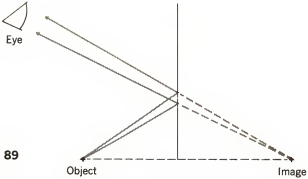


FIGURE 89

Since a small part of any mirror, plane or not, can always be considered as a plane mirror, images produced by mirrors can be found geometrically by using the two laws above.

As an example, let us consider the image formed by a large, plane mirror. Each ray of light traveling from the object to the eye obeys the laws of reflection stated in the preceding paragraph. (Naturally, only rays of light directed so as to strike the eye contribute to the image you see.) As is shown in Fig. 89, the eye sees light rays which appear to emerge from an object as far behind the mirror as the actual object is in front of the mirror. This type of image is known as a *virtual image,* since the rays of light do not actually pass through the image points, but only appear to do this. A careful drawing of rays from an extended object, such as your face, will show that the right and left sides are interchanged. We say then that the image is *perverted*. Through experience, we adjust to this when we use a mirror to shave or fix our hair. As can be seen from the diagram, the image formed by a plane mirror is upright and the same size as the object.

Aside from the plane mirror considered above, mirrors in the shape of a portion of a sphere are often used. An example is the magnifying mirror often used in shaving. Suppose that light from a distance source strikes such a mirror. In this case the rays of light will all be parallel. It is found that to a good approximation all of the parallel rays are reflected so as to pass

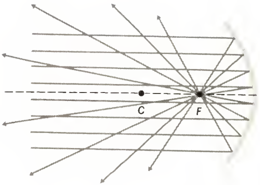
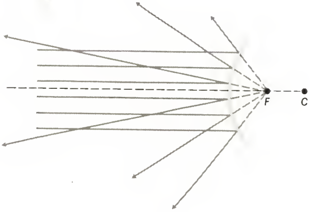


FIGURE 90

FIGURE 91



through a single point, known as the *focus* of the mirror. This is shown for a *concave mirror* in Fig. 90. It can be shown further that the focal length f is one half the radius of curvature R of the mirror. In a similar way, parallel rays of light are reflected from a *convex mirror* as if they came from a focal point, as is shown in Fig. 91. As before, the focal distance is one half the radius of curvature of the mirror.

Suppose that an object is not a great distance from the mirror and we would like to construct graphically the image which the mirror produces. A ray from the object which is parallel to the axis of the mirror is reflected as if that ray came from a very distant object. A ray which strikes the center of the mirror is reflected, by symmetry, just as it would be from a small plane mirror located at the center of the actual curved mirror. The point at which these two rays intersect is then the image point corresponding to the original object point. (It can

be shown that to a good approximation all rays from a given object point pass through the same image point.) The two special rays described above are shown for both a concave and a convex mirror in Fig. 92. In the case of the example with the concave mirror, the image is *real,* since the light rays actually pass through the image points. Furthermore, the image is *inverted, perverted,* and *magnified.* (This is not always the case with a concave mirror.) When a convex mirror is used, the image is always virtual, erect, perverted, and smaller, as is shown in the diagram.

It is possible for a concave mirror to give a magnified, erect, and perverted image. This situation is illustrated in Fig. 93. A shaving mirror, for instance, would be used in this way. The only requirement is that the object be closer to the mirror than its focal length.

In Sec. 5.3 we discussed refraction of a light ray by a prism. A simple lens usually has surfaces which are portions of a sphere, rather than the flat surfaces of a prism. If light from a

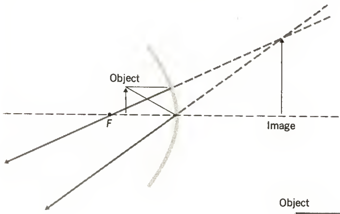


FIGURE 92

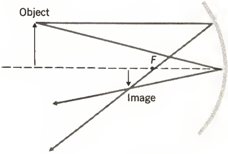


FIGURE 93

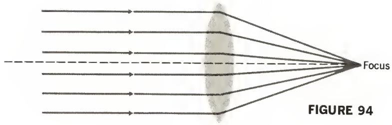


FIGURE 94

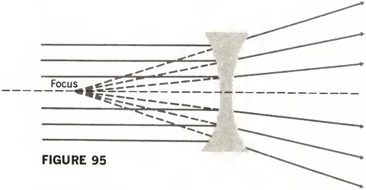


FIGURE 95

distant object, such as the sun, strikes a lens, the rays of light will be parallel to the axis of the lens. In Fig. 94 various parallel rays are shown striking a *converging lens.* To a good approximation, the rays all pass through a single point after passing through the lens. This point is called the *focus* of the lens.

In a similar way, when parallel rays of light strike a *diverging lens,* after passing through the lens they all appear to come for a point known as the *focus.* The situation is shown in Fig. 95. The reader should note that a converging lens has a real focus, since the rays of light actually pass through this point, while a diverging lens has a virtual focus, since the rays do not pass through the focal point.

We will now consider image formation when the object is not very far from the lens. For either type of lens, we can draw a line from a point on the object parallel to the axis of the lens. After passing through the lens, this ray will pass through the focus of a converging lens or appear to come from the focus of a diverging lens. A second ray we can draw from the point on the object is

one which passes through the center of the lens. By symmetry, this ray will not be deviated by the lens. The point at which the two special rays described in this paragraph intersect is then the image point corresponding to the original object point. It can be shown that other rays from the same point on the object also pass through the same image point to a good approximation. Examples of the construction of images by this graphical method are given in the following paragraphs.

Suppose that an object is situated as shown in Fig. 96 with respect to the lens. The two special rays discussed in the preceding paragraph allow us to determine the location of the image as shown in the diagram. We see that the image is inverted, real, and larger than the object. If we made a three-dimensional diagram, we would find also that the image is perverted (turned left-for-right).

If the object is moved closer to the lens than in the preceding example, the ray diagram becomes the one shown in Fig. 97. Here we find that the image is virtual, upright, and larger than

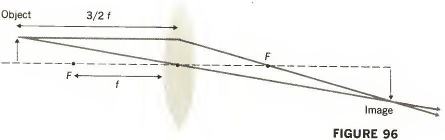


FIGURE 96

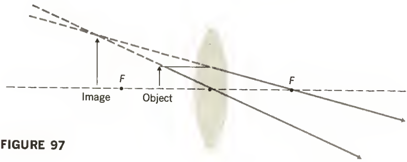


FIGURE 97

the object. In this case the image is not perverted. When we use a converging lens to magnify an object in this way, we have a simple *magnifying glass.* These are commonly used by coin and stamp collectors, for instance, and also as an aid to reading scientific instruments.

We will now turn our attention to the image formed by a diverging lens. We will again use the two special rays which allow us to locate the image easily. These rays and the image formed are shown in Fig. 98. In this case, we see that the image is virtual, upright, and smaller than the object. Also, the image is not perverted. If the object were moved to a different distance from the lens, the size and position of image would change, but the image would still be virtual, erect, smaller and not perverted. Thus, a diverging lens never gives a real image or magnification.

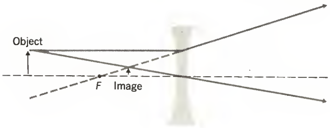


FIGURE 98

Because materials show dispersion (variation of the index of refraction with wavelength), different wavelengths emitted by the object do not focus at exactly the same point. Thus, the image is formed at different distances for different wavelengths. This is known as *chromatic aberration.* In addition, a spherical surface does not focus rays from all parts of an extended object at the same distance from the lens. This also causes blurring of the image. This type of lens error is known as *spherical aberration.* In expensive optical instruments these aberrations can be compensated for quite well by using a composite lens constructed of a number of simple lenses made from different types of glass.

A simple optical instrument which nearly everyone has used is the *camera.* A simple camera is shown in Fig. 99. As the diagram shows, a real image of the object is focussed on the

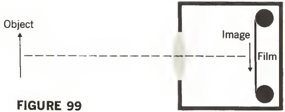


FIGURE 99

film by a single converging lens. If a sharper image is wanted, the single lens is replaced by a combination of lenses, in order to correct for the aberrations discussed in the preceding paragraph. With any lens, the image can be made sharper by using only the central part of the lens. This is achieved by having an adjustable circular diaphragm. If we close the diaphragm to a small diameter we get a sharper picture, but this allows less light to reach the film. The *f-number* of a lens is defined as the ratio of the focal length to the diameter of the lens opening. Thus, a large f-number means that the diaphragm is small, so that a better image is obtained, but also that less light passes through the lens. In general, it is best to use a camera set at the largest f-number which admits sufficient light to expose the film properly.

Another familiar optical instrument is the *astronomical* telescope. As is shown in Fig. 100 two converging lenses are used. Light from a distant object passes through the objective lens and produces a real image, I_1. This image is examined and magnified by the eyepiece lens so that the eye sees the virtual

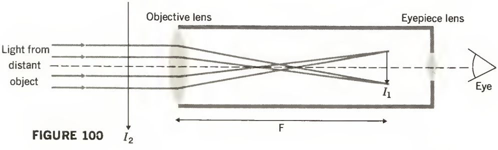


FIGURE 100

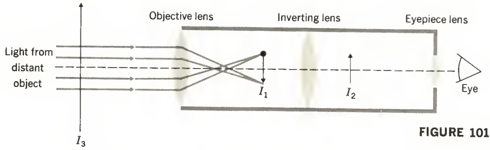


FIGURE 101

image, I_2. The chief disadvantage of this instrument is that the final image is inverted. This is not really a handicap when you are looking at stars, but is quite annoying when you are watching a horse-race, for instance. This can be overcome by introducing a third converging lens between the objective and eyepiece lenses, as is shown in Fig. 101. The only purpose of this third lens is to invert the image a second time, so that the final image is upright. Such an instrument is called a *terrestrial telescope* or *spyglass*. When we introduce the third lens, the barrel of the telescope has to be longer than before. This is the chief disadvantage of this device.

Another common optical instrument is the *microscope*. As is shown in Fig. 102, two converging lenses are used. The object

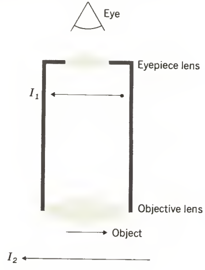


FIGURE 102

is placed close to the objective lens, which produces an enlarged, real image. This image is then examined and further magnified by the eyepiece lens. The final image is inverted, but anyone using a microscope quickly learns to compensate for this, so this is not really a major disadvantage.

Most large telescopes used by astronomers have a concave mirror which forms a real, inverted image. This image is then further magnified by an eyepiece lens. While mirrors show spherical aberration, all wavelengths are reflected the same, so that chromatic aberration is eliminated. Another advantage is that only one surface has to be carefully ground to shape. Finally, imperfections, such as air-bubbles, often occur in large lenses, while only the front surface of a mirror is used. For these and other reasons, all very large telescopes are built around concave mirrors. The largest now in operation is located at Mount Palomar, California, with the mirror having a diameter of 200 inches or $16\frac{2}{3}$ feet.

SUMMARY

We began this chapter by discussing properties which all waves have. Thus, we defined the amplitude of a wave as the maximum value of the disturbance from its equilibrium or static value. The speed with which a given part of the disturbance passes a certain point is known as the velocity of the wave. If the wave is repetitious, we say that the number of such repetitions per second at a fixed point is the frequency of the wave and the time interval between two repetitions is the period of the wave. Similarly, for a repeating wave the distance between the nearest points where the motion is the same is called the wavelength of the wave. Various relations between these quantities were then discussed. Finally we distinguished between transverse waves, in which the motion of the transmitting material is at right angles to the velocity of the wave, and longitudinal waves, in which the motion of the material is parallel to the velocity of the wave.

As a common example of a wave which happens to be longitudinal we considered sound waves in air. This led us to discuss

standing waves and musical instruments. Light waves are also familiar; these waves were used as an example of a transverse wave. Since light waves differ from other electromagnetic waves only in their frequency and wavelength, other types of electromagnetic waves were also described. These will be treated in more detail later.

A property of waves which is not true for particles is that two waves may cancel one another while two particles cannot. This phenomenon of interference was therefore discussed next, since it provides a way of deciding whether a certain effect is caused by waves or particles. Interference experiments which show that both sound and light are caused by waves were used as examples. Another characteristic of transverse waves, but not of longitudinal waves, is that they show directional properties or can be polarized. Experiments which prove that light waves show polarization effects and are thus transverse waves were described. Similarly the failure of sound waves to be polarized shows that they are longitudinal waves.

Because of its practical importance, we next studied the formation of images by mirrors and lenses. The camera, telescope, and microscope were used as examples of these ideas.

PROBLEMS

1 *If the amplitude of a simple wave is cut in half, by what factor is the energy carried by the wave changed?* ANS.: $\frac{1}{4}$.

2 *If we wish to increase the energy carried by a wave by a factor of 100, by what factor should we change the amplitude of the wave?*

3 *Calculate the period of a sound wave of frequency 40 vibrations per second.* ANS.: 2.5×10^{-2} *sec.*

4 *Water waves strike a beach every 5 sec. What are the period and frequency of these waves?*

5 *A typical frequency in the FM radio band is 10^8 cycles per second. Compute the wavelength of these waves.* ANS.: *3 m.*

6 *Repeat problem 5 for a frequency of 10^6 cycles per second, which lies in the AM radio band.*

7 *An organ pipe produces a wavelength of 5 m in air. Take the speed of sound in air as 330 m/sec and compute the frequency of these waves.*
ANS.: *66 cps.*

8 *Compute the wavelength of a sound wave of frequency 10,000 cycles per second.*

9 *Compute the value of the longest wavelength which can exist as a standing wave on a cello string 1 m long.* ANS.: *2 m.*

10 *A fundamental frequency produced by a piccolo is 2000 vibrations per second. What is the fourth harmonic of this note?*

11 *Thunder is heard 10 sec after the lightning flash is seen. How far away did the lightning strike?* ANS.: *3300 m.*

12 *Compute the wavelength of an x ray of frequency 10^{18} cps.*

13 *Two loudspeakers are 3 ft apart. When you stand 4 ft directly in front of one of the speakers, the sound intensity is very low. Calculate the probable wavelength of the sound from the loudspeakers.*
ANS.: *2 ft, $\frac{2}{3}$ ft, $\frac{2}{5}$ ft, etc.*

14 *You stand 20 ft from the vertical brick wall of a building. A point source of sound of wavelength 10 ft is moved slowly from the building directly toward you. Describe the changes in sound intensity received by your ear as the source of sound approaches you.*

15 *Refer to Problem 14. Calculate a location of the source such that would expect to receive maximum intensity at your ear.*
ANS.: *Source 5, 10, or 15 ft from wall.*

16 *Refer to Problem 14. Calculate a location of the source such that you would expect to receive minimum intensity at your ear.*

17 *Longitudinal earthquake waves have a speed of 7 km/sec, while transverse earthquake waves have a speed of 4 km/sec. If the time interval between the arrivals of the two waves at your laboratory is 10 min, how far away did the earthquake occur?* ANS.: *5600 km.*

18 *Signals from two synchronized radio transmitters are received with a time interval of 10^{-3} sec between their times of arrival. Calculate how much farther you are from one transmitter than from the other. (This provides the basis for a system of navigation known as* Loran.)

19 *An electromagnetic wave is sent out by a transmitter, reflected by a target, and received 10^{-4} sec later at the location of the transmitter. How far away is the target? (This is the principle upon which* radar *works.)*

ANS.: 1.5×10^4 *m.*

20 *Explain qualitatively why an approaching train whistle sounds higher in frequency than the same whistle when it is at rest with respect to you. (This is known as the* Doppler effect.)

21 *By what factor must the amplitude of a wave be changed for the energy carried by the wave to be increased by a factor of 9?* ANS.: *3.*

22 *If the amplitude of a wave is quadrupled, by what factor is the energy carried by the wave changed?*

23 *Calculate the period of a sound wave of frequency 15,000 vibrations per second.* ANS.: 6.67×10^{-5} *sec.*

24 *Calculate the frequency of a sound wave which has a period of 1 millisecond.*

25 *A radio transmitter broadcasts at a frequency of 15,000 cycles per second. Take the velocity of radio waves to be 3×10^8 m/sec and compute the wavelength of these waves.* ANS.: *20 km.*

26 *Some radar sets used during World War II used wavelengths of 55 cm. Compute the frequency of these electromagnetic waves.*

27 *Standing on a dock you observe wave crests passing you on the water every 2 sec. If the distance from one crest to the next following is 100 ft, what is the velocity of these waves?* ANS.: *50 ft/sec.*

28 *If the velocity of sound in air is 1100 ft/sec, what is the wavelength of sound waves of frequency 500 cycles per second?*

29 *Repeat Problem 28 for a frequency of 50 cycles per second.*

ANS.: *22 ft.*

30 *Compute the value of the longest wavelength which can exist as a standing wave on a violin string 2 ft long.*

31 *Refer to Problem 30. If the string is bowed properly, the third harmonic may be excited. What is its wavelength?* ANS.: *1.33 ft.*

32 *Refer to Problem 30. If the velocity of waves along the violin string is 2000 ft/sec, what frequency is associated with this wavelength?*

33 *Refer to Problem 32. What wavelength does this violin string produce in air, if the velocity of sound in air is 1100 ft/sec?* ANS.: *2.2 ft.*

34 *How long must an organ pipe closed at one end be if its fundamental wavelength is to be 10 ft?*

35 *Refer to Problem 34. If the velocity of sound in air is 1100 ft/sec, what is the fundamental frequency of this organ pipe?* ANS.: *110 cps.*

36 *Refer to Problems 34 and 35. What is the frequency of the second overtone of this organ pipe?*

37 *Take the velocity of sound in air as 1100 ft/sec. Compute the length of an organ pipe closed at one end which will have a fundamental frequency of 20 cycles per second* ANS.: *13.8 ft.*

38 *Light has a velocity of* 2×10^8 *m/sec in a certain glass. What is the index of refraction of this glass?*

39 *If the index of refraction of diamond is 2.42, what is the velocity of light in this material?* ANS.: 1.24×10^8 *m/sec.*

40 *An object is located 25 cm from a concave, spherical mirror of radius 20 cm. Locate the image graphically, and give its character (erect or inverted, real or virtual, magnified or diminished, perverted or not).*

41 *Refer to Problem 40, but let the object be 5 cm from the mirror.*
ANS.: *10 cm back of mirror, erect, virtual, diminished, perverted.*

42 *Refer to Problem 40, but let the mirror be convex.*

43 *Refer to Problem 41, but let the mirror be convex.*
ANS.: *3.33 cm behind mirror, erect, virtual, diminished, perverted.*

44 *An object is located 20 cm from a converging lens of focal length 15 cm. Locate the image graphically and determine its character.*

45 *Refer to Problem 44, but let the object be 10 cm from the lens.*
ANS.: *30 cm in front of lens, erect, virtual, enlarged, not perverted.*

46 *Refer to Problem 44, but let the lens be diverging.*

47 *Refer to Problem 45, but let the lens be diverging.*
ANS.: *6 cm in front of lens, erect, virtual, diminished, not perverted.*

48 *An object 2 cm tall is placed 4 cm from a converging lens of focal length 5 cm. Find graphically the size of the image.*

49 *Repeat Problem 48 for a diverging lens of the same focal length.*
ANS.: *1.11 cm.*

50 *An object is placed 6 cm from a converging lens of focal length 5 cm. Find graphically the magnification of the image.*

51 *Repeat Problem 50 for a diverging lens of the same focal length.*
ANS.: *0.455.*

DISCUSSION QUESTIONS

1 *The human ear responds only to a range of frequencies between approximately 20 cycles per second and 15,000 cycles per second. List some sources of sound which produce frequencies below the limit of human hearing (infrasonic) and frequencies above the limit of human hearing (ultrasonic).*

2 *Refer to question above. Discuss any applications of infrasonic and ultrasonic sound.*

3 *Discuss any evidence that the speed of sound is the same for all wavelengths.*

4 *Discuss how a bugler can produce different frequencies, even though the length of the air column of a bugle is fixed.*

5 *The speed of sound in air is about 1100 ft/sec and the speed of light is about 186,000 miles/sec. If you divide the time in seconds between the instant you see the lightning bolt and the instant you hear the thunder by five you will learn the approximate distance away of the lightning bolt in miles. Explain why this is so.*

6 *A stone is dropped down a deep well. The time between releasing the stone and hearing it splash in the water is measured. Discuss how from this measurement you could determine the depth of the well.*

7 *The speed of light is very great. Discuss possible methods for measuring it.*

8 *Light can be reflected from a polished surface. Does this tell us whether light is a wave or a stream of particles?*

9 *Since light is a wave, why do we not observe interference effects when two lights are turned on in the same room?*

10 *Repeat the previous question for two persons talking in a room.*

11 *Is it possible to observe interference effects when two loudspeakers operated from the same amplifier are used in a room?*

12 *A number of boomerangs could be thrown so as to pass through a horizontal rail fence but not to pass through a vertical rail fence. Discuss whether or not this proves that boomerangs are transverse waves.*

13 *A beam of light consists of waves of a single wavelength and frequency. Can you think of a way of changing the wavelength of this light without changing its frequency?*

14 *Discuss whether you think that the human ear is sensitive to frequency or wavelength.*

15 *White light is a mixture of wavelengths (colors). Why do oil slicks on rain puddles show colors?*

16 *Why do various objects have definite colors, even when they are illuminated by white light which contains all colors?*

17 *Give an example of polarization of light used quite commonly in life.*

18 *Is the sensation you perceive proportional to the intensity of light striking your eye or of sound striking your ear?*

19 *Discuss singing in a shower stall in terms of standing waves.*

20 *If the velocity of light in a material varies with wavelength, does this mean that the index of refraction of the material is not constant?*

21 *If you wished to examine a small object using a lens, would you choose a converging or diverging lens?*

22 *A diverging lens always produces a virtual image. Think of ways using an additional lens or mirror so that a diverging lens produces a real image.*

23 *Discuss how you might measure the focal length of a diverging lens.*

24 *The* Galilean telescope *or* opera-glass *consists of a converging objective lens, and a diverging eyepiece lens. Its advantage is that it produces an upright image. Sketch a typical ray diagram for this instrument.*

CHAPTER SIX

ATOMIC PHYSICS

6.1 The electron

During the nineteenth century the basic laws of electromagnetism were discovered. In this period applications, such as communication and power transfer by electrical means, began to transform our society. These applications did not depend on knowing whether electricity was carried by individual particles or was carried by a continuous fluid. In this section we will discuss the evidence for believing that electrical effects are caused by discrete particles or corpuscles.

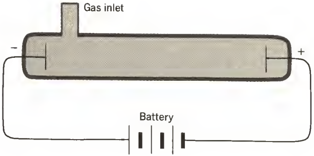


FIGURE 103

An important field of research during the second half of the nineteenth century was the study of electrical discharges through gases at low pressure. A typical piece of equipment would consist of two metal plates (electrodes) inside of a glass tube in which the gas pressure and nature of the gas could be varied. An arrangement of this sort is shown in Fig. 103. The experimenter can vary the gas pressure and the applied voltage, among other variables. If the gas pressure is low and the voltage high, we observe the colored discharge which is still used today in "neon" signs. (The color of the discharge depends on the gas or mixture of gases used.) If the pressure is reduced further, the color mainly disappears and we observe that the glass of the tube glows green. Furthermore, the discharge will now cause certain materials, such as zinc sulfide, to fluoresce or glow. (This is used today in the picture tubes of television sets.) Since this discharge appears to begin at the negative electrode (*cathode*) of the tube and is more or less independent of the location of the positive electrode (*anode*), it is known as a *cathode ray* and a tube using it is known as a *cathode-ray tube.* This interesting radiation was the subject of many investigations.

As early as 1859 Julius Plücker, a German mathematician and physicist, observed that cathode rays could be deflected by a magnetic field. From this we conclude that they are similar to an electric current. Also, from the direction of the deflection of the beam of cathode rays in either a magnetic or electric field it was found that they were negatively charged. Other experiments showed that the cathode-ray beam carried momentum and could exert a force. From the above we would guess that cathode rays consist of a stream of negatively charged particles.

In 1897 an English physicist, Sir J. J. Thomson, performed quantitative experiments in which he subjected beams of cathode rays to electric and magnetic fields. From his data he could calculate the ratio of the charge to the mass of these cathode-ray particles. He found the same value for this ratio, regardless of the gas used, the material of the electrodes, or the values of pressure and voltage he used. The conclusion from Thomson's experiments is that cathode rays consist of negatively charged particles all with the same ratio of charge to mass. The name *electron* (Greek for *amber*) is now used for these particles. Here we should point out that Thomson's work did not show that the cathode-ray particles were identical, since his experiments only showed that all of them had the same ratio of charge to mass. Naturally, one would hope that nature would be that simple, but the experiments of Robert Millikan were needed to show that indeed this was the case.

An important application today of cathode-ray tubes is the television kinescope, on which the picture is displayed. A typical tube of this sort is shown in Fig. 104. One of the most useful

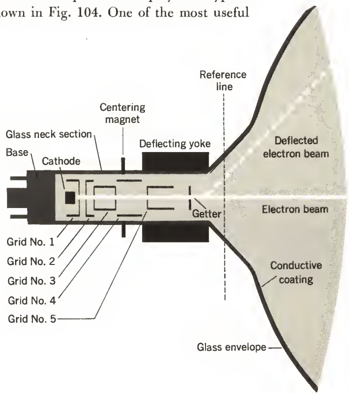


FIGURE 104

instruments in the science laboratory is the cathode-ray oscilloscope. A typical tube used in this apparatus is shown in Fig. 105. The reader should note that in the case of the television tube the electron beam is deflected magnetically, while in the case of the oscilloscope tube the electron beam experiences an electrostatic force. However, in both cases the electron beam paints a picture on the phosphorescent screen. In the case of the oscilloscope, the picture may be of a wave, showing its amplitude, wave form, or some other property.

Millikan, an American physicist, began a series of experiments in 1909 to measure the electron charge independent of the mass of the particle carrying it. Small drops of oil floating in air are observed with a microscope as shown in Fig. 106. From the speed at which they fall through air the size and mass of the drops can be computed. If now an electric charge Q is given to a drop and an electric field E is applied, the drop will experience an additional force given by $F = QE$. From measurements on the altered speed of the drop, the magnitude of the charge Q can be found. With luck the charge on a given drop can be changed a number of times, with each charge being measured. Also, charges on different drops can be measured. In all of his experiments Millikan found no charge which was very far from being an integral multiple of 1.6×10^{-19} coulombs. For instance, no charge was observed which was 2.2×10^{-19} coulombs. From

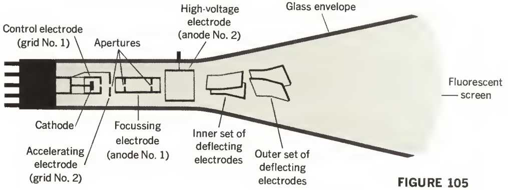


FIGURE 105

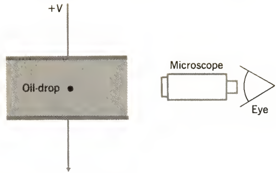


FIGURE 106

these extensive observations we conclude that electrical charge only occurs in multiples of 1.6×10^{-19} coulombs. No experiment has disproved this statement. Since Thomson's work had already shown that all cathode-ray particles (electrons) had the same ratio of charge to mass, they must therefore be identical. Later we will see that there are positive charges in nature in addition to the negative electrical charge studied by Thomson and Millikan. All such charges are found to be integral multiples of the electron charge, although they may be carried by particles with quite different masses. Thus, today we believe that there is a unit of electrical charge, equal in magnitude to the electron charge, from which all charges in nature are constructed. In brief, we say that electric charge is *quantized,* since it can only occur in definite, discrete amounts.

6.2 Black-body radiation

A body which absorbs completely all of the radiation hitting it is called a black body. Since the mechanism by which a body is a perfect absorber or black body is clearly unimportant, all such bodies have the same properties. One way to construct a black body experimentally is to line a cavity with lampblack and drill a small hole in the wall of the cavity. Any radiation from the outside which enters the hole has a very small chance of getting out again after it hits the interior of the cavity, because of the high absorption of lampblack. The small amount of radiation which is reflected from the lampblack has a very slight chance of being aimed at the hole and emerging. Thus, the hole in the cavity is almost a perfect absorber of radiation incident on it.

Reflecting enclosure

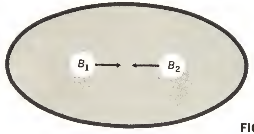


FIGURE 107

It is easy to show that all black bodies at the same temperature have the same properties as regards the emission of radiation. Consider two black bodies at the same temperature enclosed by a perfectly reflecting cavity, as shown in Fig. 107. Let us assume that body 1 transfers more energy to body 2 than body 1 receives from 2. This assumption violates the second law of thermodynamics, since there is a transfer of energy between the bodies without a difference in their temperatures. From this we conclude that each body has the same emission characteristics, so that neither body gains energy from the other. Thus, all black bodies have both the same absorption properties as well as the same emission properties when their temperatures are the same. This should not be surprising, since a perfect absorber is an ideal body, so that we could hardly expect one ideal body to have better emission characteristics than another.

When the energy radiated by a black body is measured, the relative emission depends on wavelength in the way shown in Fig. 108. If the temperature of the body is raised to a higher value, the upper curve of Fig. 108 is found. Wilhelm Wien found that the product of the wavelength of maximum emission and the temperature of the black body was a constant. In the form of an equation we can write that the wavelength of most intense emission, $L_{\max}$, and the absolute temperature T of the black body are related by:

$$L_{\max}T = 2.90 \times 10^{-3} \text{ m-°K} \tag{87}$$

Also, Stefan and Boltzmann observed that the total radiation over all wavelengths of a black body was proportional to the

fourth power of the absolute or Kelvin temperature of the black body. In terms of an equation, the total radiation per unit of time per unit of area, E, is related to the Kelvin temperature T of the black body by the equation:

$$E = (5.67 \times 10^{-8})T^4 \text{ (watts/m}^2\text{)} \qquad \textbf{(88)}$$

The task of theory at the turn of the century was to predict the curves shown in Fig. 108 and the relations of Eqs. 87 and 88.

By 1900 classical theory had been shown to be incorrect in accounting for the experimental laws of black-body radiation. Planck took a new direction in his theory of black-body radiation. First he assumed that the radiation came from oscillators in the walls of the cavity. Further, he assumed that an oscillator of frequency f could only have an energy which was some integral multiple of hf, where h was a constant to be determined. In terms of an equation, Planck's first hypothesis is that the only possible energy E_n of such an oscillator would be given by one of the values:

$$E_n = nhf \qquad (n = 1, 2, 3, \ldots) \qquad \textbf{(89)}$$

Thus, the energy of an oscillator is quantized. In addition, Planck assumed that if the energy of such an oscillator changed

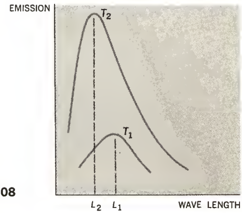


FIGURE 108

from one characterized by a particular value of the integer n, say n_1, to one characterized by a different value, say n_2, the difference in energy would be emitted in a single burst or unit of radiation, known as a *photon* or *quantum*. The change in energy of the oscillator, $E_1 - E_2$, would be related to the frequency f' radiated by the equation:

$$n_1hf - n_2hf = E_1 - E_2 = hf' \tag{90}$$

The constant h in Eqs. 89 and 90 is now known as *Planck's constant*, and has the value 6.63×10^{-34} joule-sec.

When Planck applied the ideas above in combination with classical theory, he was able to account for all of the experimental laws of black-body radiation. Since only one value of the constant h could be chosen, it is astonishing that a single choice could lead to complete agreement with the curves of Fig. 108 and the relations given by Eqs. 87 and 88. Even the numerical values of the constants of Eqs. 87 and 88 could be determined quite accurately from Planck's theory. His ideas were the origin of the whole theory of atomic structure which we will discuss later in this section.

In Sec. 6.1 we saw that electrical charge occurred only in certain definite amounts, which were integral multiples of the charge of the electron. We said then that electrical charge was quantized, since it only occurred in discrete amounts. According to Planck's theory of radiation, radiant energy is also only emitted in discrete amounts, known as quanta or photons. Within a few years the idea that nature is continuous was attacked by these theories. We will soon see that the absorption of radiation is also discontinuous, and later in this chapter we will discuss the evidence for the view that matter is made up of individual particles. Today the facts seem to indicate overwhelmingly that all of the physical universe is corpuscular. A sandy beach may appear as continuous when seen from an airplane, but a closer study shows its granular nature. The same idea applies to the structure of matter, which is always found to be corpuscular or quantized when viewed closely enough.

6.3 The photoelectric effect

When light strikes certain metals, such as zinc, the metal emits a small electric current. While this effect was noticed by Hertz in 1887, it was first investigated carefully by Hallwachs and is often named after him, but is more commonly referred to as the photoelectric effect. In this section we will discuss this interaction between light and electricity. We will see also that light is absorbed in photons or quanta, just as Planck's theory of black-body radiation showed that light is emitted in the same way.

A typical arrangement for studying the photoelectric effect is shown in Fig. 109. The glass bulb is evacuated to a high vacuum, so that nothing interferes with the electric current between the two electrodes. During the last decade of the nineteenth century it was shown that the electric current travelling from the negative to the positive electrode was carried by particles identical with the particles observed by Thomson as cathode rays. Thus, the photoelectric current consists of electrons, which are called *photoelectrons*. Evidently in some way light can eject electrons from the negative electrode. These negative particles are then drawn to the positive electrode and collected there. The ammeter indicates the flow of electrons per second or the photoelectric current.

Not surprisingly, the photoelectric current in amperes is proportional to the intensity (watts/m^2) of the incident light

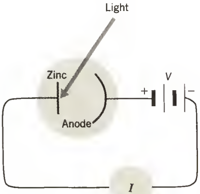


FIGURE 109

over a very wide range of intensities. This is important in applications of photoelectric tubes, since it is desirable that the output of the device (current) be proportional to the input (light intensity). This proportionality is what we might expect classically.

If we measure the energy of the most energetic photoelectrons, we find a surprising result. The maximum energy of a photoelectron produced from a given surface by light of a given wavelength is independent of the intensity of the light. Normally we would expect that if we raise the light intensity and thus hit the surface with more energy per second, at least some of the photoelectrons would get more energy. This result is clearly not something which we would have predicted.

If we vary the frequency of the incident light, keeping the intensity the same, we find that the maximum energy of the photoelectrons varies in a straight-line or linear way as is shown in Fig. 110. (Actually, the first accurate measurements were made by Millikan, 1910–15.) Furthermore, below a certain critical or threshold frequency for a given surface, no photoelectrons are emitted, regardless of how large the intensity of the incident light might be. Classically, the maximum energy of the photoelectrons should depend on the energy hitting the surface and not depend at all on the frequency of the incident light, but here we find just the reverse. Clearly, classical theory cannot account for the experimental facts of photoelectricity.

In 1905 Einstein solved the puzzle described above by combining the ideas that electricity occurs in the form of particles

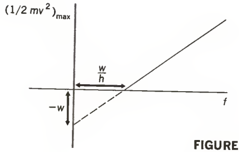


FIGURE 110

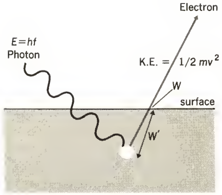


FIGURE 111

(electrons, in this case) and that electromagnetic radiation is transmitted in units of energy called quanta or photons. Since Planck had shown that radiation is emitted as quanta, Einstein assumed that radiation is also absorbed as quanta. This would mean that all of the energy of a given quantum would be given to a single electron. If this amount of energy was large enough, the electron might escape the metal and eventually be collected by the positive electrode.

Einstein's ideas concerning the nature of photoelectric emission are shown schematically in Fig. 111. The incident photon of frequency f brings in an amount of energy given by $E = hf$, where h is Planck's constant derived from studies on black-body radiation. All of this energy is then given to a single electron near the surface of the metal. Part of this energy, W', is used up in the electron's trip to the surface of the metal, and additional energy W is used up when the electron passes through the forces existing at the surface of the metal. (If it were not for these forces at the surface of the metal, electrons would leak out spontaneously.) The remaining energy is then retained by the electron in the form of kinetic energy after it has emerged from the metal. If we apply the idea of conservation of energy to this situation we have the following equation:

$$E = hf = W' + W + \tfrac{1}{2}mv^2 \qquad \textbf{(91)}$$

Unfortunately, Eq. 91 cannot be verified, since the amount of energy W' that an electron loses in reaching the surface of the metal varies from one electron to another. Some electrons may not be aimed directly at the surface or may start too far from the surface to reach it at all. Other electrons may be at the surface to begin with or may lose no energy in reaching the surface, so that $W' = 0$ for these electrons. The values of W' for other electrons will lie between these two extremes. However, there is no way to tell what the various values of W' will be for a given metallic surface.

The situation is simpler if we consider only those electrons for which $W' = 0$. As we see from Eq. 91, these electrons emerge from the surface with maximum kinetic energy for the particular frequency and metal used. We can write then:

$$hf = W + (\tfrac{1}{2}mv^2)_{\text{max}} \tag{92}$$

If we solve Eq. 92 for the kinetic energy $(\frac{1}{2}mv^2)_{\text{max}}$ of the most energetic electrons, we find

$$(\tfrac{1}{2}mv^2)_{\text{max}} = hf - W \tag{93}$$

If we examine Eq. 93 we see that it explains the puzzling dependence of the maximum energy of photoelectrons on frequency. Since light intensity does not appear in Eq. 93, the maximum energy of the photoelectrons should be independent of the light intensity, as is observed. If the energy hf is less than W, there should be no emission of photoelectrons at all, since negative kinetic energy has no physical meaning. Finally, if hf is greater than W, Eq. 93 predicts a straight-line relation between the frequency f and the maximum kinetic energy of the photoelectrons. Thus, all of the features of the photoelectric effect are explained by Einstein's theory. From the success of this theory we conclude that electromagnetic radiation is both emitted and absorbed in units of energy of magnitude hf, which are called quanta or photons. We will make further use of this idea later in this chapter.

6.4 Atomic spectra and structure

As we discussed in Sec. 5.3, light is an electromagnetic wave. Light which the human eye can see is called *visible light.* The average human eye responds to wavelengths from about 4×10^{-7} meters (violet) to about 7×10^{-7} meters (red). Wavelengths which are somewhat longer than those of visible light are called *infrared* wavelengths. Similarly, wavelengths somewhat shorter than visible light are called *ultraviolet* wavelengths. Later we will discuss much shorter wavelengths, which are known as x rays.

Each element when it is in the gaseous state and properly excited emits a variety of wavelengths. These wavelengths are as characteristic of the element as your fingerprints are of you. Thus, we can identify the various elements present in a given material by observing the wavelengths of light which the material emits. For instance, the yellow color we see in a gas flame when a pot on the stove boils over is caused by the sodium in the salt used to flavor the food. Similarly, when we burn driftwood, we observe various pretty colors due to the minerals which the wood has absorbed from seawater. Finally, a tube filled with neon gas at low pressure when excited electrically gives off the reddish color of the element neon which we observe in "neon" signs. In general, whenever we add energy to a substance in large enough quantities, we excite the elements in the material to emit their characteristic wavelengths.

The study of the wavelengths emitted by various elements (or combinations of elements into molecules) is known as *spectroscopy.* The collection of wavelengths given off by an element or molecule is known as its *spectrum* (plural: *spectra*). After the spectroscopist has measured the wavelengths of a certain spectrum, it is the job of the theorist to account for these observations on the basis of hypotheses which seem plausible and not in disagreement with basic theories of nature.

The first spectrum to be analyzed experimentally was that of hydrogen, the simplest element. The visible spectrum of hydro-

FIGURE 112

gen is shown in Fig. 112. In 1885 Balmer noticed that all of the wavelengths in the visible spectrum of hydrogen could be calculated from the simple formula of Eq. 94, which is shown below:

$$L = \frac{3.646 \times 10^{-7}\ \text{m}}{1 - 4/n^2} \qquad (n = 3, 4, 5, \ldots) \tag{94}$$

When we substitute a value of n from 3 upwards into Eq. 94, we obtain one of the wavelengths of the visible spectrum of hydrogen. The quantity n, which can take on only integral values of 3 and greater, is known as a *quantum number*. In fact whenever we have a quantity which can take only certain values and none in between we refer to it as a quantum number or quantized variable.

Somewhat later Lyman found a series of wavelengths emitted by hydrogen in the ultraviolet which could be expressed by a similar formula in which n can take on only integral values from 2 upwards. Similarly another group of wavelengths emitted by hydrogen can be calculated also from an equation similar to Eq. 94 in which n takes on only integral values of 4 or more. We would certainly feel that this similarity in three *spectral series* of hydrogen could not be a coincidence.

Observations on the spectral series of other elements showed that these could be accounted for by equations which were modifications of Eq. 94. As was the case for hydrogen, the wavelengths of a given spectral series could all be computed by allowing some quantity to take on successive integral values, such as 2, 3, 4, ... Furthermore, the constants involved in these equations were all approximately the same, regardless of the element studied. The simplicity and generality of these equations suggests that the

same mechanism must be responsible for the wavelengths emitted by all elements. The job of the theorist was to construct a single theory which would account for the wavelengths characteristic of all elements.

We will now take a sidetrack and discuss possible models of atomic structure. In addition to explaining the facts of spectroscopy, any theory of atomic structure must agree with other observations on atoms and molecules. Since atoms are known to be electrically neutral, an atom must consist of equal amounts of positive and negative charges. In addition, the theory must account for the chemical properties of the elements, as well as other properties. These requirements, of course, make the problem much more difficult.

The answer was given in 1912 by the British physicist Ernest Rutherford and his coworkers, Geiger and Marsden. In order to probe the atoms in a metal foil, they used high-speed particles emitted by substances such as radium. (In the following chapter we will find that these particles are helium ions and have a positive charge.) When one of these particles strikes a fluorescent screen of zinc sulfide, for instance, a flash of light is emitted. Thus, by observing the screen we can determine exactly where each of these particles has hit the screen. The experimental arrangement is shown schematically in Fig. 113. When the metal foil is not in place, all of the particles from radium strike a small spot at the center of the fluorescent screen. However, when the metal foil is put in the path of the beam, quite a few flashes occur well away from the center of the screen. From this observation we conclude that these particles have been deflected or scattered by the material in the metal foil.

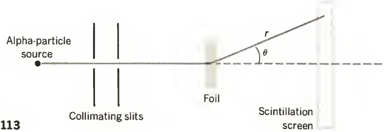


FIGURE 113

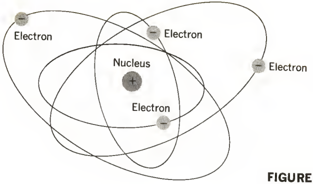


FIGURE 114

Since the particles emitted by radium were known to have a positive charge of two units, it was possible to calculate the amount of deflection such a particle would be given by a particular model of the atom. By this time Thomson had proposed as a model of the atom a sphere of positive electricity in which were embedded negative electrons. This model was generally accepted. By analogy, this was known as the plum-pudding model of the atom. Thomson's model predicts that the particles emitted by radium would be deflected only one or two degrees. Since deflections through large angles were observed fairly frequently, this model of the atom did not explain Rutherford's observations.

Because of the failure of the Thomson model of the atom to agree with the experimental data, Rutherford was led to propose a different model. He suggested that an atom might consist of a small, massive core, called the *nucleus,* which would be positively charged and comprise most of the mass of the atom. Travelling about the nucleus in various orbits would be a number of light, negatively charged electrons. A hypothetical picture of such an atom is shown in Fig. 114. This model of the atom did predict large deflections of the particles emitted by radium, and the measurements were in complete agreement with Rutherford's theory. By 1913 this solar-system model of the atom was well established. With some refinements, this is still the model of the atom which we use today.

6.5 Atomic energy levels

In 1913 Niels Bohr of Denmark used the Rutherford model of the atom to explain the spectrum of atomic hydrogen. According to his ideas an atom of hydrogen consists of an electron revolving in a circular orbit about a singly charged nucleus, which we call a *proton*. Furthermore, the electron could only be in one of a number of possible orbits. This is a break with classical theory, which would allow the electron to be in an orbit of any size. The various orbits, possible or allowed, were to be characterized by a quantum number which could take on the values 1, 2, 3, ... Some of these orbits are shown in Fig. 115, although not drawn to scale. The energy of the electron was also determined by the quantum number of the orbit in which it happened to be. As we saw earlier in connection with Planck's theory of black-body radiation, the electron could only have certain definite energies and no others.

In our discussion of the photoelectric effect we saw that all of the energy of a photon was given to a single electron. Bohr suggested that the reverse should also be true. In this case, if the energy of an electron in a hydrogen atom decreased, all of this change in energy would be emitted in the form of a single photon, with an energy hf and a frequency f. Mathematically, if the energy of an electron decreases from a value E_1 to a value E_2 the energy

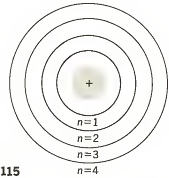


FIGURE 115

and the frequency of the photon emitted would be given by the equation:

$$hf = E_1 - E_2 \qquad \textbf{(95)}$$

Conversely, the absorption of an energy hf by an atom should raise the energy of the electron by the same amount given in Eq. 95. Thus, if we could compute the possible values of the energy of the electron, we could predict all of the frequencies which could be emitted by the atom.

Bohr's theory was completely successful when applied to the hydrogen atom. Not only did the theory account for the known spectral series of hydrogen, but it also predicted the existence of additional series in the infrared. Measurements on these previously unknown spectra agreed excellently with the predictions. Thus, Bohr's theory met both requirements of a successful theory: the theory accounted for known facts and also correctly predicted previously unknown facts which were then observed experimentally.

The various energy levels of hydrogen are shown in Fig. 116. Normally, the atom would exist in the state of lowest energy, corresponding to $n = 1$. This is known as the *ground state* of the atom. If energy is given to the atom, perhaps by a collision with another atom or by electrical means, the energy of the electron is raised to one of the higher states, known as *excited states.* After a very short time the electron will return to its lowest energy state by jumping to successively lower energy states. During each such jump or transition, a photon is emitted with a frequency given by Eq. 95. In any such transition, we speak of the higher energy state as the *initial state* and the lower energy state as the *final state.* All of the frequencies emitted in transitions ending on the same final state are said to be part of a particular spectral series. In Fig. 116 some of the transitions which lead to the Lyman and Balmer series are indicated.

When Bohr's theory is applied to slightly more complicated atoms, such as helium with its two electrons, the theory does not give results in agreement with experiment. For more than a decade attempts were made to extend the theory to atoms aside

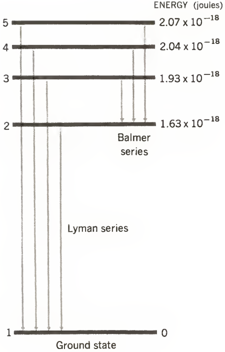


FIGURE 116

from hydrogen, but without success. In 1926 Werner Heisenberg and Erwin Schrödinger, German physicists, independently formulated the correct theory, which is known as *wave mechanics* or *quantum mechanics*. The idea of atomic energy levels was retained, but a much different way of calculating these energy levels was proposed. After the various energy levels of an atom are known, the possible frequencies which the atom can emit (or absorb) are calculated using Eq. 95. As in the case of hydrogen, the frequencies which result from transitions from various initial states to a single final state form a spectral series.

Aside from predicting the energy levels and thus the frequencies emitted by atoms, this new form of mechanics also accounted for other atomic properties. For instance, the formation of molecules from two or more atoms was successfully explained. In addition, the similarities in the properties of various elements—which led to the construction of the periodic

table—was given a theoretical foundation. By using this new theory, the existence of a few strongly magnetic materials, such as iron, could also be understood. The new mechanics has been applied successfully to a wide variety of problems in atomic physics, and one can feel quite sure that it will solve all problems involving atoms. In practice, however, the mathematical work becomes too difficult for an exact solution to be found, so that various approximation methods have to be used. Aside from small errors due to the use of necessary approximations, the theory has been successful in all of its applications to the properties of atoms.

An interesting example of the idea of atomic energy is the *maser*, which is an acronym standing for *M*icrowave *A*mplification by the *S*timulated *E*mission of *R*adiation. Let an atomic system have an energy E_1 in its lowest energy state and an energy E_2 in its first excited state. Normally, most of the atoms will be in the lowest state. Suppose that electromagnetic waves of frequency f given by $hf = E_2 - E_1$ strike the substance. Atoms in the lower state will absorb a photon and be raised to the high state, as shown in Fig. 117. Similarly, atoms in the higher state will be stimulated to emit a photon of the same frequency, as shown in Fig. 118. Since at equilibrium there are more atoms in the lower state than in the higher state, the net effect on a beam

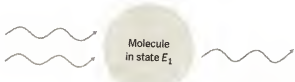


FIGURE 117

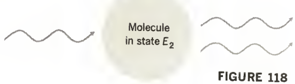


FIGURE 118

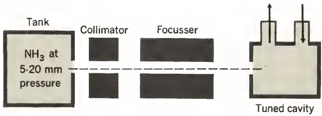


FIGURE 119

of electromagnetic waves passing through the substance is absorption of energy.

In order for stimulated emission of radiation to be the dominant process, the number of atoms in the higher state must be made larger than the number in the lower state. This was first achieved in 1954 by Townes, Gordon, and Zeiger using the apparatus shown in Fig. 119. They used as an atomic system the ammonia molecule, NH_3, but masers using other materials have since been operated. Ammonia molecules diffuse from the tank and are collimated into a narrow beam by slits. The focuser consists of parallel rods with alternating electric charges, as shown looking toward the ammonia tank in Fig. 120. The effect of the electric field produced by the focuser is to push molecules in the higher state toward the axis of the apparatus and to push molecules in the lower state away from the axis of the apparatus. Thus, most of the molecules in the lowest state do not enter the tuned cavity. The dominant process inside the cavity now becomes the stimulated emission of radiation, if the proper frequency enters the cavity.

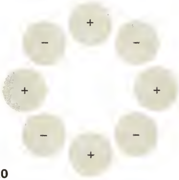


FIGURE 120

Suppose that a small amount of radiation of the correct frequency enters the cavity. These photons stimulate molecules in the higher state to emit photons of the same frequency. If the cavity is tuned to resonate at this frequency, standing waves are set up in the cavity which stimulate numerous molecules to emit photons. We can now take energy out of the cavity, which makes the device an amplifier. In the case of ammonia the frequency is approximately 25 kilomegacycles, which lies in the range of frequencies known as microwaves. This explains the first word in the title of the device. When other materials are used in place of ammonia, other frequencies not in the microwave region can be amplified, but the original term is used for all devices of this sort except the laser, or light-amplifying maser, which uses a ruby rod.

6.6 X rays

As we mentioned in Sec. 6.1, a popular branch of research in the late nineteenth century was the conduction of electricity through a gas at low pressure. In 1895 Wilhelm Roentgen was working with apparatus similar to that shown in Fig. 103. It had been known that cathode rays inside the tube would make certain minerals fluoresce. This effect is still used in the picture-tube of television sets and in other applications. Strangely, however, Roentgen noticed that a fluorescent screen *outside* of the tube also lit up when the gas in the tube was excited electrically. Evidently some previously unknown radiation was passing through the walls of the tube and causing fluorescence. For lack of a better name, Roentgen called this new type of radiation x rays.

In the gaseous discharge used by Roentgen in his original work on x rays, electrons moving toward the positive electrode knocked additional electrons off the gas molecules. Thus, a single electron could produce many electrons through collisions. This process is known as an *electron avalanche*. Basically, in

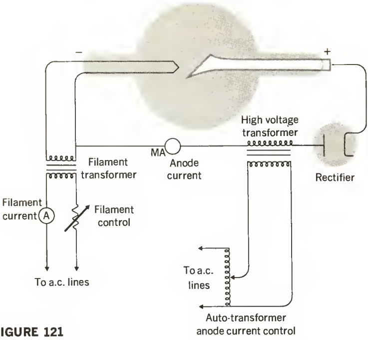


FIGURE 121

order to produce x rays, we need a source of electrons, a high voltage to accelerate the electrons, and a target which emits the x rays after being struck by the electrons. Of course, all of these elements were in the discharge tubes used by Roentgen.

Unfortunately, if we wish to increase the number of electrons in a gas-filled tube (and thus the x-ray intensity), we must apply a larger voltage to the positive electrode. This has the effect of making the x rays more penetrating, which may not be desired. In 1913 W. D. Coolidge devised an x-ray tube which is similar to those used today. This tube is shown in Fig. 121. The Coolidge x-ray tube is evacuated to the best vacuum possible. The source of electrons is a heated wire, and the rate of emission of electrons from this source can be adjusted by varying the filament current. Since the positive high voltage can be varied independently, the limitations of the gas-filled tube are overcome. When we vary the filament current, we vary only the quantity of x rays produced per second. If we change the positive high voltage, we change only the penetrating power of the x-ray beam.

In a masterpiece of careful investigation Roentgen discovered the basic properties of x rays within a matter of months. Aside from the production of fluorescence in certain materials, x rays were also found to affect photographic plates and to produce ionization in gases. Both of these properties are still used in detecting x rays. He found also that materials differ in their transparency to x rays; this difference is the basis for using them to study the internal structure of objects which are opaque to visible light. (Within three months x rays were being used by physicians as aids in the setting of bone fractures.) In generating x rays Roentgen observed that radiation was emitted whenever high-speed electrons struck a target. Furthermore, the penetrating power of the x rays was increased if the material of the target had a high atomic number or the voltage applied to the tube was increased. Other properties of x rays were also reported by Roentgen, such as the fact that they travel in straight lines and are not affected by a magnetic field.

Since x rays are not influenced by a magnetic field, they cannot be charged particles. They must therefore be either neutral particles or waves. In 1912 Laue and the Braggs gave the answer to this question by showing that x rays showed destructive interference, which can be explained only by a wave theory. Furthermore, their methods allowed the measurement of x-ray wavelengths, which turn out to be in the approximate range of 10^{-12} to 10^{-10} meters. Thus, x-ray spectroscopy became possible.

When measurements are made on the wavelengths emitted by the target of a typical x-ray tube, two types of spectra are observed as is shown in Fig. 122. There is a *continuous spectrum,* having a wide range of wavelengths with no gaps. This spectrum depends mainly on the voltage applied to the tube and the thickness of the target. Suppose that an electron hits an atom in the target and is slowed down from an energy E_1 to an energy E_2 in the collision. The change in energy of the electron is then emitted in the form of a single photon of frequency f given by the equation:

$$hf = E_1 - E_2 \tag{96}$$

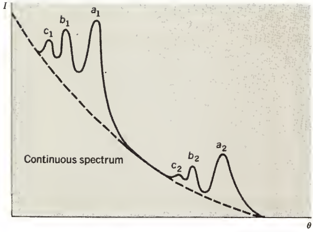


FIGURE 122

Equation 96 is the same as Eq. 95, but the mechanism of the change in energy is different. Since electrons may undergo collisions ranging from direct hits on atoms to glancing collisions, electrons will lose different amounts of energy. This explains the continuous range of emitted frequencies and wavelengths. A typical electron will give up its energy in several collisions, each collision producing a quantum of radiation.

If the voltage applied to the tube is high enough, for a given target we also observe certain wavelengths characteristic of the material of the target, which are marked *a, b, c* on the diagram. This *bright-line* or *discrete spectrum* is similar to the spectra discussed in Sec. 6.4, except that x-ray wavelengths are much smaller than visible wavelengths. We can analyze a sample of material by using it as a target in an x-ray tube, since each element in the target will produce its own characteristic wavelengths.

By analogy with the theory of atomic spectra presented in Sec. 6.5, we would guess that these discrete wavelengths would be produced when the energy of an electron in an atom decreases from one allowed value to a lower allowed value. In fairly complex atoms the quantum theory predicts that groups of electrons exist in shells or layers at different distances from the nucleus of the atom. Electrons in a shell close to the nucleus are held

more closely than electrons in distant shells. Thus, more energy is required to excite an electron in an inner shell than to excite an electron in an outer shell. The energies involved are therefore much higher than the energies involved in exciting the outer electrons, which are responsible for the emission of photons of visible light. To summarize, visible spectra are produced by energy changes in the outer electrons of an atom, while discrete x-ray spectra are produced by larger energy changes involving the inner electrons of an atom. Aside from this difference, the mechanism is the same in both cases, and either Eq. 95 or Eq. 96 can be used to find the frequency emitted after a particular change in energy.

6.7 Wave versus particle properties of matter

Earlier in this chapter we saw that radiation is emitted and absorbed as if radiation consisted of particles each with an energy $E = hf$. In 1923 Arthur Compton performed some experiments in which x rays were scattered by a solid material. He found that some of the scattered x rays had longer wavelengths than the initial beam. Furthermore, the wavelength of these scattered x rays varied with the direction in which they were scattered. This is known as the *Compton effect* and later won Compton a Nobel prize in physics.

Compton's experiment is shown in Fig. 123. The original x rays have a wavelength L, while the scattered x rays have a wavelength L' which varies with the direction of observation measured by the angle A. Furthermore, an electron is knocked out of the scattering material. The energy acquired by the electron also depends on the direction in which it travels. Since some of the energy of incident x-ray photon is given up to the electron, we can see qualitatively that the scattered or modified photon must have less energy. Thus, the frequency of the scattered photon must be less and its wavelength greater.

In order to explain his experimental results quantitatively, Compton assumed that a photon carried an energy $E = hf$ and

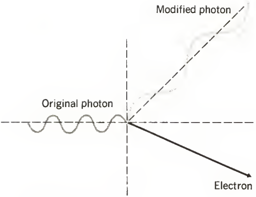


FIGURE 123

also possessed a momentum p given by $p = hf/c = h/L$. When the original photon strikes an electron in the scattering material, Compton suggested, the impact should be treated according to the usual rules of mechanics. If we consider the x-ray photon to have an energy hf and a momentum h/L, we can then apply the laws of conservation of energy and momentum, just as if we were dealing with a particle. The results of this theory gave good agreement with experiment. To summarize Compton's very important idea, a photon of wavelength L has a momentum p associated as given here:

$$p = \frac{hf}{c} = \frac{h}{L} \tag{97}$$

This seems paradoxical, as we think of a photon as a wave, but the only real test of a theory is its agreement with experiment. Since Compton's idea met the test well, we have to believe that photons can act like particles under the right circumstances.

Usually we expect to find reciprocal properties in nature. Louis De Broglie, a French physicist, suggested in 1924 that if photons carry a momentum given by Eq. 97, then particles of momentum p should have associated with them a wavelength L given by:

$$L = h/p = h/mv \tag{98}$$

This brilliant guess was verified experimentally by C. J. Davisson and L. H. Germer of the U.S.A. and by G. P. Thomson of

England. Their work showed that under proper conditions a beam of electrons showed destructive interference, which is a property of waves. Furthermore, the wavelengths observed agreed with those computed from Eq. 98. Later it was shown that heavier particles and even atoms possessed wavelengths in agreement with De Broglie's hypothesis. Today there can be no doubt that under certain conditions a wave (photon) may act like a particle and a particle may act like a wave.

The wave and particle dualities discussed earlier in this section lead to a consideration of possible measurements on a particle such as the electron. We will consider here simultaneous measurements of the position and momentum of a particle. Suppose we try to do this by using photons. If we wish to observe the position of the particle accurately, we must use photons of very short wavelength, so that diffraction effects are negligible. Unfortunately, photons of this sort will have a great frequency and momentum, so that they will knock the particle far off its original course. Thus, we would know the position of the particle quite well, but would have changed its momentum greatly both in direction and magnitude. On the other hand, to measure the velocity and momentum of the particle, we need to know the particle's positions at two instants separated by a known time interval. If we do this by using photons of long wavelength, so that we do not change the particle's momentum appreciably, then we will have only a poor knowledge of the particle's position at a given instant.

The ideas of the preceding paragraph were first expressed quantitatively by Heisenberg in 1927. If Δx is the uncertainty in the measurement of position and Δp is the uncertainty in the measurement of momentum, then we have the following relation:

$$\Delta x \, \Delta p = h \tag{99}$$

The equation above tells us that if we wish to know the value of the position of a particle very accurately, we can have only a meager knowledge of the momentum of the particle at the same instant. This is known as *Heisenberg's uncertainty principle.*

Quite soon it was shown that the uncertainty principle could be derived from quantum mechanics, and that it applied to other pairs of simultaneous measurements as well. For instance, the same uncertainty exists between our knowledge of the energy of an atom and the time at which it had this energy. Lastly, it should be stated that these uncertainty relations do not depend on the skill of the experimenter or the choice of apparatus. The uncertainty principle simply reflects the statistical nature of matter, and no ingenuity can overcome this. An analogy might be the attempt to predict the toss of a single coin one time. This is bothersome to those philosophers who believe in causality, since future events are not exactly governed by the present and predictable by mathematically exact laws.

The reader is probably now wondering how something can act in one way under certain conditions and in another way under other conditions. To the author it seems that the words "wave" and "particle" are meaningful only for large-scale objects, which we can observe directly. It appears that trying to extend these ideas to the realm of very small objects simply fails. Here it should be noted that quantum mechanics predicts this wave-particle duality in nature. In extreme cases we can use either the wave idea or the particle concept, but in most cases we deal with a mixture where the entity cannot be described as either a wave or a particle. This is an example of the danger of trying to transfer the use of words from one area of experience to another.

*6.8 Relativity

Until the end of the nineteenth century it was believed that there was an absolute space and an absolute time. The laws of nature would have their simplest forms when expressed in the coordinates and time of this absolute system. Furthermore, by taking the proper measurements it would be possible to determine our speed through this space. In this section we will see

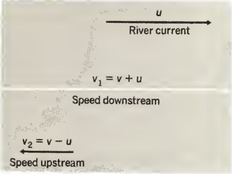


FIGURE 124

that these ideas had to be given up, since measurements did not agree with the theory. Then we shall discuss the changes proposed by Einstein which led to the theory of special relativity.

Suppose that we were in a boat which could travel with a speed v relative to still water. Movements in two directions are shown in Fig. 124 if we are located on a river with a current of speed u. From the diagram we see that our speed downstream, v_1, would be:

$$v_1 = v + u \tag{100}$$

Similarly, if we traveled up the river, our speed v_2 would be:

$$v_2 = v - u \tag{101}$$

From our measurements of v_1 and v_2, we could compute the speed of the boat relative to the water, v, and the speed of the water itself, u, by solving Eqs. 100 and 101 simultaneously. Alternatively, by measuring v and u we could calculate v_1 and v_2, the speed of the boat relative to the banks of the river.

In an analogous experiment using light waves, A. A. Michelson and E. W. Morley tried to measure the speed of the earth relative to absolute space. Since the speed of the earth in its orbit about the sun is about 18 mi/sec while the velocity of light is about 186,000 mi/sec, it was thought that the observation of the speed of the earth through absolute space should be relatively simple. Regardless of the time of day or the season of the year, no change in the velocity of light due to the earth's assumed motion

was observed. Other optical or electromagnetic experiments performed later by other scientists also could not detect any effect by the earth's motion, even though the theories then in use predicted results which should have been clearly observable. From these unsuccessful experiments, we might conclude that there is something wrong with the idea that the earth's motion will affect observations in a way analogous to the way in which motion of a river will affect measurements made aboard a boat moving on the river.

Einstein came to the conclusion that ordinary ideas about motion were wrong and that there is no such thing as absolute space. Since the only motion which is physically measurable is motion relative to a material body, we must confine ourselves to such motions. Thus, the vague idea of motion relative to an absolute space was given up. As a substitute for motion relative to an absolute frame of reference, Einstein proposed in 1905 that motions be considered only relative to an *inertial system.* We may define an *inertial system* as a frame of reference in which Newton's laws of motion discussed in Sec. 2.5 hold. Experimentally there is found to be such a system. Any other system moving with respect to this first system with constant relative velocity will also be an inertial system, as we will now show. A particle which obeys Newton's first law and moves subject to no forces with constant velocity will still have a constant, although different, velocity relative to the second system, and will thus obey Newton's first law in the second system. Similarly, a particle subjected to a certain acceleration relative to the first system will have the same acceleration relative to the second system, since the systems differ only by a constant relative velocity. Thus, Newton's second law will hold in the second system, and the same force will be considered as acting on the particle in either system. Finally, since the forces acting on a particle are the same in both systems, Newton's third law will hold in the second system if it holds in the first system. In passing it should be pointed out that an inertial system is not an accelerated system, and all inertial systems differ from one another only by a constant relative velocity.

From the considerations of the paragraphs above, Einstein was led to propose the two basic postulates of the theory of special relativity, which are as follows:

1. All observers located in inertial systems measure the same value for the velocity of light. This postulate might be considered to be based on the unsuccessful attempts to measure an observer's speed through absolute space, since the observer's apparent motion did not change his measured value of the velocity of light.

2. There is no preferred inertial system for the expression of physical laws. By this we mean that all physical laws and principles will have the same mathematical form, regardless of the particular inertial systems used to express them. If a particular law has the form $P = QR$ when expressed in terms of the coordinates and time of system S, it will have the form $P' = Q'R'$ when expressed in terms of the coordinates of another inertial system S′.

When Einstein's principles of relativity given above are applied to physical theories, it is found that changes in their classical forms are required. A length L moving past an observer at a speed v will be found to have a smaller value than the same length would have in a system in which it is at rest. If we let the length of an object in a system in which it is at rest be L' and the speed of light be c, we find that:

$$L = L'\left(1 - \frac{v^2}{c^2}\right)^{1/2} \qquad \textbf{(102)}$$

From Eq. 102 we see that a meterstick moving past an observer is measured to be shorter than an identical meterstick at rest in the observer's frame of reference. Similarly, a clock moving past an observer at a constant speed v is observed to run slow by the same factor, namely,

$$T = \frac{T'}{(1 - v^2/c^2)^{1/2}} \qquad \textbf{(103)}$$

We interpret Eq. 103 to mean that we would observe on our clock a greater interval between events than a clock moving past us with speed v would indicate, so that we say that the moving clock runs slow compared to ours, even if both were of identical construction.

At this point the reader might raise the point as to whether or not a moving length or clock does indeed have properties different than an identical object would have if it were at rest relative to the observer. The answer is, of course, that motion has no effect whatsoever on the actual properties of an object but only on those observed. However, the "real" length of a measuring stick plays no part in physical theory, in which only measured quantities are of interest. By this we mean that an observer must assign a smaller value to the length of a rod moving past him than he would assign to an identical rod at rest in his system. Similar remarks apply to time observations. Everyday velocities are very small compared to the velocity of light, so that we do not notice these relativistic effects. Relativistic predictions are well confirmed in the laboratory, and thus we cannot reject them merely because our intuition fails us.

Since length and time measurements on high-speed particles yield different values than we might expect, it should not be surprising that the same thing occurs in the case of mass. If we observe a particle with mass m_0 at rest in our system, the mass m of an identical particle moving past us with a constant speed v is found to be:

$$m = \frac{m_0}{(1 - v^2/c^2)^{1/2}} \tag{104}$$

Thus, as the speed of a particle increases, its mass increases also, as is shown in Fig. 125. Since a greater force is needed to accelerate a particle of greater mass, as the mass of a particle gets very large further acceleration becomes more difficult. For this reason it is impossible for the speed of a particle even to reach the speed of light, much less exceed it. For similar reasons the speed of a signal can not be greater than the speed of light, which therefore becomes an upper limit on speeds observable in the physical universe.

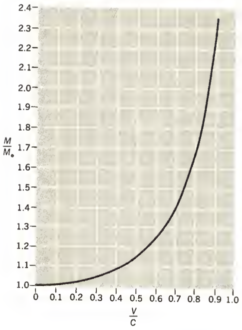


FIGURE 125

When we investigate the effect of relativity on energy, we find new relations too. In particular we find that mass can be converted into energy and vice versa, using the relation:

$$E = mc^2 \tag{105}$$

In Eq. 105 if mass is measured in kilograms and the velocity of light is expressed in meters per second ($c = 3 \times 10^8$ m/sec approximately), then the energy comes out in joules. From Eq. 105 we see that quantities such as mass and energy, which used to be considered as entirely different, are really as closely related as the two faces of a coin. Our conservation principles must now conserve the two quantities together, using Eq. 105 to convert between the two concepts. This equivalence between mass and energy is thoroughly confirmed in numerous experiments in nuclear physics, of which the most notable is the so-called "atomic bomb." In the case of the *nominal atomic bomb,* defined as a nuclear bomb capable of releasing energy equivalent in explosive power to 20,000 tons of TNT, approximately 3

grams of mass are converted into energy. Aside from this spectacular example, most nuclear phenomena would be inexplicable without the principle of the equivalence of mass and energy. Here it should be pointed out that the source of the enormous amount of energy radiated by our sun is the conversion of mass into energy at the rate of about 4.4 million tons per second. From the foregoing the reader should conclude that the results of the theory of relativity are well confirmed experimentally. Intuitive judgments, therefore, are no basis for deciding that the theory is not correct.

SUMMARY

Evidence for the existence of a unique particle now called the electron was presented. As was the case in the so-called "discovery" of America, the electron had been waiting to be observed for years. However, the quantitative experiments of Thomson (1897) and Millikan (1910–1915) showed that a single particle could explain all of the various phenomena which we now associate with the electron.

During the same period many observations were made on sources of electromagnetic radiation, particularly those which radiate in the visible region of wavelengths. The study of the radiation from hot solids led Planck to propose that radiation was emitted in discrete amounts, called quanta or photons, each of which had a distinct energy. A few years later Einstein explained the photoelectric emission of electrons by postulating that radiation was absorbed in exactly the same way, with the entire energy of a given photon being given to a single electron. Thus, the idea that radiation is absorbed in discrete amounts was established.

When the radiations from various atoms are studied, it is found that a given element emits only certain characteristic wavelengths. Rutherford's work in 1912 established that an atom consists of a massive, positively charged core surrounded by a cloud of electrons. This led Bohr and others to suggest that the electrons associated with an atom could have only certain energies, and that if the energy of an

electron changed, the difference in energy would all be emitted in the form of a photon with exactly this amount of energy. Later this idea was extended to include the various possible energy states of molecules. It was also seen that x rays, which are radiations of much higher frequency than visible light, originate also by changes between possible energy states of an atom.

Another important theory of the early twentieth century was the special theory of relativity proposed by Einstein. Because of failures of earlier theories, he suggested that the ideas of absolute space and time were meaningless. Instead, only motion relative to a material frame of reference could have physical meaning. Furthermore, Einstein postulated that the laws of nature would have the same mathematical form when expressed relative to any reference system moving with constant velocity. From these ideas he deduced that an observer would measure different values of length, mass, and time for objects moving past him than he would for identical objects at rest with respect to him. In addition, he predicted that mass could be converted into energy and vice versa, as has been abundantly verified in the "atomic" bomb and other phenomena involving nuclei.

During the 1920's, several other new ideas came along to upset the traditionalists. Particles, such as electrons, also acted like waves under certain circumstances. Conversely, photons could act like particles. In fact, the very words themselves ceased having a definite meaning when men studied the sub-microscopic nature of things. Causality was upset when Heisenberg presented his theory that the simultaneous measurement of two quantities with perfect accuracy was not always possible, even in principle. Thus, definite ideas about nature which had been held for centuries had to be modified.

PROBLEMS

1 *Compute the electrical force on an oil droplet carrying a charge of 3.2×10^{-19} coulombs when it is located in an electric field of 2000 volts/m.* ANS.: 6.4×10^{-16} *nt.*

2 *Refer to Problem 1. Compute the mass and weight of such a droplet which would be in equilibrium under the action of such an upward electrical force and the force of gravity.*

3 *Compute the wavelength at which electromagnetic radiation is most intense from a human body at a temperature of 40° C.*

ANS.: 9.27×10^{-6} *m.*

4 *Refer to Problem 3. If the surface of a human body is 2 m^2, compute the total radiation from the human body, treating it as an ideal radiator.*

5 *The most intense radiation from our sun occurs at a wavelength of about 6×10^{-7} m. Compute the surface temperature of the sun.*

ANS.: 4.83×10^{4}°*K.*

6 *An electric heater is rated at 100 watts. If its surface area is 300 cm^2, compute its temperature.*

7 *Assume that a typical frequency of FM-radio is 10^8 cycles/sec. Compute the energy of a photon of this frequency.* ANS.: 6.63×10^{-26} *joules.*

8 *Compute the frequency of a photon which has an energy of 10^{-9} joules.*

9 *Refer to Problem 8. Compute the wavelength of such a photon.*

ANS.: 1.99×10^{-25} *m.*

10 *A photon has a wavelength of 6×10^{-7} m. Compute the energy of such a photon in joules.*

11 *A photon of frequency 6×10^{14} cycles/sec hits a surface which requires an energy $W = 2 \times 10^{-19}$ joules for a photoelectron to escape. Calculate the energy of the most energetic photoelectrons emitted by this material.* ANS.: 1.98×10^{-19} *joules.*

12 *Refer to Problem 11. Repeat the problem for the case that the incident frequency is 6×10^{13} cycles/sec.*

13 *Calculate the wavelengths of the three longest wavelengths of the Balmer series of hydrogen.*

ANS.: 6.56×10^{-7} *m*; 4.86×10^{-7} *m*; 4.34×10^{-7} *m.*

14 *Calculate the value of the shortest wavelength in the Balmer series.*

15 *If the longest wavelength in the Balmer series of hydrogen is 6.56×10^{-7} m, calculate the energy change which produces this wavelength.* ANS.: 3.01×10^{-19} *joules.*

16 *Calculate the minimum energy which must be added to an unexcited hydrogen atom for the longest wavelength of the Balmer series to be emitted.*

17 *Refer to Fig. 116. Calculate the least energy which must be given to an unexcited hydrogen atom for the electron to be removed from the atom entirely.* ANS.: 2.17×10^{-18} *joules.*

18 *Calculate the frequency of an x ray of wavelength* 10^{-11} *m.*

19 *Refer to Problem 18. Calculate the energy of a photon of such a wavelength.* ANS.: 6.63×10^{-23} *joules.*

20 *Calculate the energy change so that a photon of wavelength* 10^{-13} *m will be produced.*

21 *Calculate the momentum associated with a photon of wavelength* 10^{-13} *m.* ANS.: 6.63×10^{-21} *kg-m/sec.*

22 *Calculate the wavelength associated with a particle of mass 1 kg moving at a speed of 10 m/sec.*

23 *Assume that the speed of the earth in its orbit around the sun is* 10^{-4} *times the speed of light and that the diameter of the earth is 8000 mi. Compute the contraction of the size of the earth as viewed by someone on the sun.* ANS.: *2.53 in.*

24 *Calculate the amount of energy released when 3 gm of mass disappear.*

25 *Calculate the speed of an electron if its mass is to be twice its rest mass.* ANS.: 2.60×10^{8} *m/sec.*

26 *An oil droplet has a mass of* 10^{-17} *kg and carries an electrical charge of* 4.8×10^{-19} *coulombs. Compute the value of the electric field which will hold this droplet in equilibrium against the force of gravity.*

27 *A furnace operates at a temperature of 1500°C. Compute the value of the wavelength which is most intensely emitted by this furnace.* ANS.: 1.64×10^{-6} *m.*

28 *If the door of the furnace described in Problem 27 measures 30 × 60 cm, compute the amount of the radiation emerging (in watts) when the door is fully open.*

29 *If the surface temperature of the sun is 6000°K, compute the wavelength of most intense emission by the sun.* ANS.: 4.83×10^{-7} *m.*

30 *At the earth's surface the radiation received by sunlight is 1.3 kilowatts/m^2 on a surface at right angles to the sunlight. Treat the human body as a black body of area 0.75 m^2. Compute the energy received in 0.5 hour at noon by a sunbather.*

31 *Compute the energy of a photon of electromagnetic radiation which has a wavelength of 3 cm, as is used in radar.* ANS.: *6.63×10^{-24} joules.*

32 *Assume (incorrectly) that sound waves of frequency 500 cycles/sec are quantized as electromagnetic waves are. Compute the energy of such a sonic photon.*

33 *Compute the frequency of a photon which has an energy of 10^{-18} joules.* ANS. *1.51×10^{15} cps.*

34 *Refer to Problem 33. Compute the wavelength of this photon.*

35 *A photon has a wavelength of 10^{-8} cm. Compute the energy of this photon in joules.* ANS.: *1.99×10^{-15} joules.*

36 *Light of wavelength 5×10^{-7} m strikes a surface which requires an energy of $W = 1.5 \times 10^{-19}$ joules for a photoelectron to escape. Compute the energy of the most energetic photoelectrons emitted by this material.*

37 *Refer to Problem 36 and repeat the problem for radiation of wavelength 5×10^{-6} m.* ANS.: *None emitted.*

38 *The longest wavelength which will cause a certain material to emit photoelectrons is 4.5×10^{-7} m. Compute the value of the photoelectric work-function of this material in joules.*

39 *Calculate the wavelengths of the three longest wavelengths emitted in the Balmer series of hydrogen from Eq. 94.*
ANS.: *6.56×10^{-7} m; 4.86×10^{-7} m; 4.34×10^{-7} m.*

40 *Calculate the value of the shortest wavelength emitted by hydrogen.*

41 *Calculate the value of the energy change when mercury emits its resonance line at a wavelength of 2.536×10^{-7} m.*
ANS.: *7.87×10^{-19} joules.*

42 *Calculate the least energy which must be given to an unexcited hydrogen atom for the longest wavelength of the Lyman series to be emitted.*

43 *Some cosmic rays have energies of 10^{-10} joules. Calculate the frequency of such a photon.* ANS.: *1.51×10^{23} cps.*

44 *Refer to Problem 43. Calculate the wavelength of such a photon.*

45 *Refer to Problems 43 and 44. Calculate the momentum of such a photon.* ANS.: 3.32×10^{-19} *kg-m/sec.*

46 *If a particle moves with a speed of* 10^7 *m/sec, what must its mass be if its associated wavelength is to be 1 cm?*

47 *A particle of mass 5 kg has an associated wavelength of* 10^{-30} *m. What is the velocity of the particle?* ANS.: 1.32×10^{-4} *m/sec.*

48 *Calculate the contraction in length of a train 1 mi long moving at a speed of 90 mph.*

49 *Refer to Problem 48. Would it be possible to measure this contraction in length, and, if so, how?* ANS.: *No.*

50 *Compute the speed at which a passing car would have an observed length half of the length which it would have when it was at rest relative to the observer.*

51 *Compute the amount of mass which has to be converted into energy for a total amount of energy equal to* 10^{20} *joules to be released.*

ANS.: *1111 kg.*

52 *Refer to Problem 51. If this amount of energy is released during* 2×10^{-4} *sec, what is the average power during this time?*

53 *An electron has a speed of* 2.9×10^8 *m/sec. What is its dynamical mass?* ANS.: 3.6×10^{-30} *kg.*

54 *Compute the speed that an electron must have in order for its dynamical mass to be equal to the rest mass of a proton which is* 1.67×10^{-27} *kg.*

DISCUSSION QUESTIONS

1 *"The color of an oxygen atom is green." Discuss whether or not this statement is physically meaningful.*

2 *In Chapter 5 we discussed interference experiments which show that light is a wave, and yet in this chapter we find that light is transmitted in packets known as photons or quanta each of energy given by* $E = hf$. *Discuss whether light is a wave or particle.*

3 *Measurements on the photoelectric effect depend greatly on the way in which the surface is prepared. Explain why this is so.*

4 *"On Monday, Wednesday, and Friday we treat the electron as a particle, and on Tuesday, Thursday, and Saturday we treat it as a wave." Does this statement have an accurate physical meaning?*

5 *Discuss the great discoveries which came about because various men studied the apparently trivial topic of the conduction of electricity through gases. Describe a number of important ways in which these discoveries have had an impact on our lives today.*

6 *What are the experimental difficulties in using Eq. 88 as a definition of temperature?*

7 *Why are the walls of a thermos bottle silvered?*

8 *Why do you think that Planck proposed the revolutionary ideas expressed in Eqs. 89 and 90?*

9 *If sound waves are quantized in the same way that light waves are, do you think this could be detected experimentally?*

10 *Describe some applications of the photoelectric effect. On what properties of this effect do these applications depend?*

11 *If you could design the best photoelectric surface for use in the camera tube of a television transmitter, what properties would your surface have?*

12 *Give examples of objects in your own life which are quantized.*

13 *An electric heater or hotplate usually glows a dull red. At what wavelengths do you think most of the energy is transmitted?*

14 *The study of the masses of the various atoms of elements is called mass spectroscopy. Explain the origin of this term.*

15 *How would you undertake to show experimentally that atoms and molecules are usually electrically neutral?*

16 *An atom of carbon has six electrons revolving about its nucleus. If you assume that the orbits increase steadily from the smallest to the largest, which electron would correspond to the earth in our solar system?*

17 *In the various spectra of a given element, a particular energy level will serve as the final state for one spectral series and for the initial states*

of a number of wavelengths in other spectral series. Discuss how this method of internal consistency is used in analyzing spectra.

18 *Discuss the appearance of a light bulb if e had the value 1 coulomb.*

19 *X rays are absorbed most effectively by elements of high atomic number, and the amount of absorption depends on the amount of the material used. For these reasons, lead, which has atomic number 82 and high density, is most commonly used to absorb x rays. Do you think it would be worthwhile to look for some other element which would be as good an absorber as lead, yet would make a lighter shield?*

20 *Discuss changes in our life if Planck's constant had the value* $h = 10^{-20}$ *joule-sec.*

21 *Discuss changes in our life if the speed of light had the value* $c = 100$ *km/hr.*

22 *Discuss changes in our life if both h and c had the values suggested in the two preceding questions.*

23 *Discuss your belief or disbelief in the theory of relativity.*

24 *Which is more fundamental, mass or energy?*

CHAPTER SEVEN

NUCLEAR PHYSICS

7.1 The discovery of radioactivity

The discovery of radioactivity was as accidental as the discovery of x rays. In 1896 the French physicist Antoine H. Becquerel was studying the phosphorescence produced when sunlight hits certain minerals. After the sunlight has been cut off, the minerals glow with a light of steadily decreasing intensity. (The only difference between fluorescence and phosphorescence is that a fluorescent material gives off its light almost instantaneously, while a longer time is required for a phosphorescent material.) From this seemingly unimportant research came the discovery of radioactivity, from which grew the larger field of nuclear physics.

Among the minerals which Becquerel was studying were some containing uranium. He found that these uranium salts would affect photographic plates, even when the plates were wrapped in black paper. Even more amazingly, photographic plates were affected by these minerals months after they had been exposed to sunlight. Furthermore, the magnitude of the effect on a photographic plate did not change with time, although the intensity of phosphorescence drops off fairly rapidly as time passes. Becquerel therefore concluded that this effect had nothing to do with sunlight and was not an example of phosphorescence, but rather was a property of uranium itself. He named this strange property *radioactivity*. In 1898 the Curies, who discovered radium, showed that a number of other elements possessed the ability to affect photographic plates, and that some of these elements were much more active than uranium. As has happened before, an unexpected result of research opened up a whole new field of human knowledge, which has had a very great effect on our lives today.

A number of experiments uncovered other properties of radioactivity. In addition to affecting photographic plates, these radiations can ionize gases, cause fluorescence, and make certain materials better electrical conductors. Thus, these radiations can be detected in a variety of ways. The intensity of radioactivity is found not to depend on the state of chemical combination of an element, the temperature of the material, or the application of an electric or magnetic field. We conclude that radioactivity must be a property of the parts of the atom which are best shielded from external influences, such as the nucleus or perhaps the innermost electrons. Finally it should be pointed out that these radiations can injure or kill living cells. While this is the basis for using these materials to destroy cancers, for instance, it is also a danger to the careless worker.

The nature of the radiations is more complex than x rays. Suppose that we put a small piece of radioactive material, such as radium, at the bottom of a hole drilled in a lead block, as shown in Fig. 126. Since lead is one of the best absorbers of radiation, emerging from the hole will be a narrow beam of

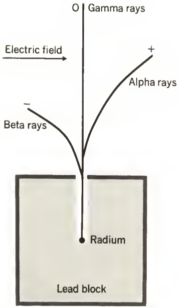


FIGURE 126

radiation. If we now apply an electric field from left to right as shown in the diagram, we will find the radiation is split into three beams, which are labeled alpha, beta, and gamma, after the first three letters of the Greek alphabet. From the directions of the deflections, we see that *alpha rays* are positively charged particles, *beta rays* are negatively charged particles, and *gamma rays* are uncharged. The three rays can also be distinguished by their penetrating powers in various materials. Alpha rays are stopped by from 5 to 10 centimeters of air, while beta rays will travel through from 2 to 10 meters of air. Gamma rays are the most penetrating, since they will pass through centimeters of lead. Finally, these three radiations differ in the ionization they produce in materials. Alpha rays produce the greatest amount of ionization, gamma rays produce very little ionization, and beta rays are intermediate in their production of ionization.

Experiments covering many years were necessary to discover the exact nature of alpha, beta, and gamma rays. In 1908 Rutherford allowed alpha rays to pass into a thin-walled tube

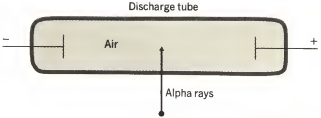


FIGURE 127

containing air as shown in Fig. 127. When at first a spark was passed through the tube, only the spectrum of air was seen. As time passed, however, the spectrum of helium also appeared more and more strongly. From this it is clear that alpha rays are ions of helium which stop inside the tube and become neutral atomic helium. Other experiments showed that in fact alpha rays are doubly charged helium ions now called alpha particles. Observations showed that beta rays are the same as ordinary electrons, except that they come from nuclei instead of the outer parts of atoms. Similarly, gamma rays are identical in their properties with x rays, except that they are produced by certain atoms rather than by an x-ray tube. Also, gamma rays usually have shorter wavelengths and thus higher energies than x rays. Since gamma-ray frequencies have no relation to the x-ray frequencies emitted by the same atom, we conclude that they are emitted when the energy of the nucleus changes. Generalizing, we state that all three of the nuclear radiations are properties of the nucleus of the atom.

As we mentioned earlier in this section, radiation passing through a gas will usually produce *ionization*. By this we mean that electrons are knocked off the gas molecules by the radiation, leaving the molecule no longer electrically neutral, so that we call it an *ion*. To produce an ion with a positive charge of one unit requires that one electron be knocked off the molecule. Thus, in the process of ionization the number of electrons freed from molecules equals the number of positive ions produced.

One device for measuring the ionization produced by radiation is the *ion chamber*, which is shown in Fig. 128. If a positive voltage is applied to the central wire, negative electrons will be

attracted toward this wire and collected by it. With a suitable instrument the magnitude of the charge carried by these electrons can be measured and from this information we can determine the number of electrons produced by the radiation. As we discussed in the preceding paragraph, the number of ions produced is equal to the number of electrons separated from the gas molecules. Finally, if we know the volume of the ion chamber, we can calculate the number of ions per unit volume produced by the radiation.

The amount of gamma rays and x rays is measured by the number of electrons or positive ions produced in a given volume. The amount of this type of radiation is defined to be one *roentgen unit* when the radiation produces approximately 2.08×10^9 electrons or positive ions in one cubic centimeter of air at 0°C and at standard atmospheric pressure. The intensity of an x-ray or gamma-ray beam is then measured in a unit such as the roentgens per second. The ionization produced by other radiations, such as alpha and beta rays, is often measured in roentgens per second also, but the effects on living cells or materials are not always the same.

Penetrating radiations can cause various effects on living animals. These radiations can cause a decrease in the number of white blood cells, loss of hair, temporary or permanent sterility,

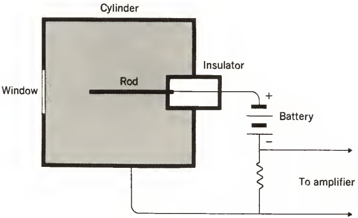


FIGURE 128

IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIII	VIII	VIII	IB	IIB	IIIA	IVA	VA	VIA	VIIA	0
								1 H 1.0080									2 He 4.003
3 Li 6.939	4 Be 9.012											5 B 10.81	6 C 12.011	7 N 14.007	8 O 15.999	9 F 18.998	10 Ne 20.183
11 Na 22 990	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 31.974	16 S 32.06	17 Cl 35.453	18 Ar 39.948
19 K 39.102	20 Ca 40.08	21 Sc 44.96	22 Ti 47.90	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.71	29 Cu 63.54	30 Zn 65.37	31 Ga 69.72	32 Ge 72.59	33 As 74.92	34 Se 78.96	35 Br 79.909	36 Kr 83 80
37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92 91	42 Mo 95.94	43 Tc (99)	44 Ru 101.1	45 Rh 102.91	46 Pd 106.4	47 Ag 107.870	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126 90	54 Xe 131.30
55 Cs 132.91	56 Ba 137.34	57 *La 138.91	72 Hf 178.49	73 Ta 180.95	74 W 183.85	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.09	79 Au 196 97	80 Hg 200.59	81 Tl 204.37	82 Pb 207 19	83 Bi 208.98	84 Po (210)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	89 †Ac (227)															

* Lanthanide series	58 Ce 140 12	59 Pr 140.91	60 Nd 144.24	61 Pm (147)	62 Sm 150.35	63 Eu 152.0	64 Gd 157.25	65 Tb 158.92	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.04	71 Lu 174.97
† Actinide series	90 Th 232.04	91 Pa (231)	92 U 238.03	93 Np (237)	94 Pu (242)	95 Am (243)	96 Cm (247)	97 Bk (245)	98 Cf (251)	99 Es (254)	100 Fm (253)	101 Md (256)	102 (254)	103 Lw (257)

FIGURE 129

mutations in the offspring, cancer, and death. The effects of repeated doses are cumulative to a great extent, although the body does recover in time. For instance, a dose of 600 roentgens taken over the entire body at one time almost certainly would be fatal, but the same dose spread out over a lifetime would probably have no observable effect. Also, since the roentgen measures the dosage per unit volume, a dose of 1000 roentgens applied to a small cancer might effect a cure, while the same dose applied to most of the body would almost certainly be lethal. After years of experience, it has been found that a worker can experience a dose over his whole body of 0.1 roentgen per day or 0.3 roentgen per week indefinitely without suffering harm. Nevertheless, unnecessary exposure to radiation should be avoided.

7.2 Laws of nuclear transmutation

If nuclei emit helium ions, electrons, and x rays, we might wonder what changes occur in the nucleus due to these processes. In Sec. 6.4 we discussed the evidence that atoms consist of nuclei with a positive charge and with most of the mass of the atom. The number of negative electrons surrounding the nucleus must be the same as the number of positive charges on the nucleus, so that the atom as a whole will be electrically neutral. This number of positive or negative charges is called the *atomic number* Z of the atom and is always a whole number or integer. Here it should be noted that the atomic number defined above is exactly the same as the chemical atomic number determined by the element's position in the periodic table, Fig. 129. If we define the mass of a carbon atom to be exactly 12 units, then it is found that the relative masses of all other atoms are very nearly whole numbers also. The nearest integer is called the *mass number* of the atom in question. The mass number of an atom is usually almost the same as the chemical atomic weight of the element. Later we will make this difference more clear. The

custom is to indicate the atomic number of an atom by a subscript preceding the chemical symbol for the element and the mass number of the atom by a superscript following the chemical symbol. If we wish to designate an atom of oxygen of atomic number 8 and mass number 16, we write ${}_8O^{16}$. Similarly, ${}_2He^4$ means an atom of helium of atomic number 2 and mass number 4. (The reader should study the periodic table of the elements shown in Fig. 129 and learn their chemical symbols, if he is not already familiar with them.) It should be noted that while the atomic number is always an integer, the mass number is only a good approximation to the mass of the atom.

If pure radium is enclosed in a sealed tube, at first a certain number of alpha particles will be observed per second. After the passage of a few days, the number of alpha particles detected per second increases greatly, and also we find beta and gamma radiation. If now the radium is no longer enclosed, but instead a stream of air flows past the radium, the air shows radioactivity. From this experiment we conclude that radium gives off a radioactive gas which is swept away by the air. This gas, which is called radon, can be studied by pumping it out of a sealed tube containing radium. Radon is found to have a characteristic optical spectrum, an atomic number of 86, and an atomic weight of nearly 222. It is therefore not the same element as radium, which has atomic number 88 and atomic weight 226. Clearly, in some way one element, radon, is produced when another element, radium, emits an alpha particle. The problem is to find the rule or law which applies to *radioactive transformations* of this kind.

Rutherford and Frederick Soddy suggested in 1912 the answer to the question raised in the preceding paragraph. They theorized that when a nucleus emits an alpha particle, the result is a new nucleus with a different mass and charge. Furthermore, both mass and charge must be conserved in the process. Since the alpha particle is a helium nucleus, symbolized by ${}_2He^4$, it has two units of charge and four units of mass. Thus, when a nucleus of radium emits an alpha particle, the resulting nucleus must be two units lower in charge (atomic number) and four units lower

in mass (mass number). In the form of an equation we can write:

$$_{88}Ra^{226} = {}_{2}He^{4} + {}_{86}?^{222}$$

However, the product of the disintegration of radium was already known to be radon, which has atomic number 86 and mass number 222. Thus, the question mark in the equation above can be replaced by the symbol for radon, Rn. We notice also that the subscripts (atomic numbers) and the superscripts (mass numbers) add up to the same value on both sides of the equation. We conclude that nuclear charge and mass are separately conserved (kept constant) in a nuclear transmutation equation.

We might now ask if the same rule applies when a nucleus emits some other type of particle, such as a beta ray (electron). Since a beta ray has a negative charge of one unit and negligible mass, we will symbolize it as $_{-1}e^{0}$. A typical example of a transmutation is shown below:

$$_{82}Pb^{214} = {}_{-1}e^{0} + {}_{83}Bi^{214}$$

In the reaction above, lead (Pb) emits a beta ray, and the product is bismuth (Bi). Again we see that the atomic numbers and the mass numbers on the two sides of the equation balance separately.

The series of transmutations producing the uranium family of elements is shown in Fig. 130. In the diagram the Greek letter α indicates that the nucleus has emitted an alpha particle, while the Greek letter β indicates that the nucleus has emitted a beta ray (electron). The energy of the particle is shown to the right of the Greek letter. The half-life of each nucleus is a concept we will discuss shortly. The archaic and obsolete symbols for the various elements are shown in parentheses; as an example $_{84}Po^{218}$ was once called RaA. The reader should notice that the series begins with $_{92}U^{238}$, which has a very long half-life, and ends with $_{82}Pb^{206}$. As alpha particles and beta rays are emitted as shown in the diagram, the various intermediate elements are produced.

To summarize the rule for nuclear transmutations, the atomic numbers and the mass numbers must be the same on each side

${}_{92}\mathbf{U}^{238}$ (U-I) 4.51×10^9 yr

α 4.18 mev

${}_{90}\mathbf{Th}^{234}$ (UX_1) 24.1 days

β 0.193 mev

${}_{91}\mathbf{Pa}^{234}$ (UX_2) 1.175 min

β 2.31 mev

${}_{92}\mathbf{U}^{234}$ (U-II) 2.48×10^5 yr

α 4.76 mev

${}_{90}\mathbf{Th}^{230}$ (Io) 8.00×10^4 yr

α 4.68 mev

${}_{88}\mathbf{Ra}^{226}$ 1622 yr

α 4.78 mev

${}_{86}\mathbf{Rn}^{222}$ 3.825 days

α 5.49 mev

${}_{84}\mathbf{Po}^{218}$ (RaA) 3.05 min

α 6.00 mev

${}_{82}\mathbf{Pb}^{214}$ (RaB) 26.8 min

β 0.65 mev

${}_{83}\mathbf{Bi}^{214}$ (RaC) 19.7 min

99.96%

β 1.65 mev 0.04%

${}_{84}\mathbf{Po}^{214}$ (RaC′) α 5.50 mev 1.64×10^{-4} sec

α 7.68 mev ${}_{81}\mathbf{Tl}^{210}$ (RaC″) 1.32 min

β 1.96 mev

${}_{82}\mathbf{Pb}^{210}$ (RaD) 19.4 yr

β 0.017 mev

${}_{83}\mathbf{Bi}^{210}$ (RaE) 5.00 days

β 1.17 mev

${}_{84}\mathbf{Po}^{210}$ (RaF) 138.4 days

α 5.30 mev

${}_{82}\mathbf{Pb}^{206}$ (RaG) stable

FIGURE 130

of the equation. This rule holds regardless of the types of particles involved or the complexity of the transmutation. In some cases four or more particles and nuclei may take part in a transmutation. Nevertheless, the equation for such a transmutation must still balance according to the rule given in this paragraph.

If we obtain a sample of radon gas and observe the number of alpha particles emitted from it, we find that the number emitted per second or *activity* decreases quite a bit within a few days. This decrease in activity is shown in Fig. 131. We define the *half-life* of a radioactive element to be the time for the activity of a sample to decrease to one-half of its initial value. In the case of radon this time is found to be about 3.83 days. The half-lives of other radioactive elements are found to range from times of less than a millionth of a second up to a time greater than a billion years.

The activity of a radioactive element is measured in terms of the number of particles it emits per second. By definition, one *curie* is 3.7×10^{10} particles or disintegrations per second. Since this is a rather large unit, the *millicurie* (3.7×10^{7} disintegrations per second) and the *microcurie* (3.7×10^{4} disintegrations per second) are more commonly used.

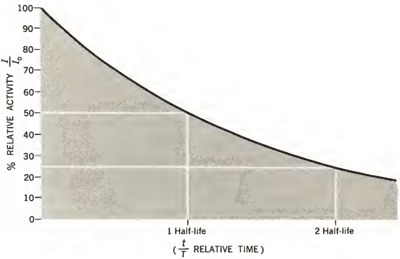


FIGURE 131

As an example, let us consider measurements made on a sample of ${}_{14}P^{32}$, which has a half-life of 2 weeks. If initially the activity of the sample is 100 millicuries, at the end of 2 weeks it will be $100/2 = 50$ millicuries. After another two weeks its activity will be $50/2 = 25$ millicuries. During every two-week period the activity of the sample will decrease by a factor of two, so that eventually we will not be able to detect it at all.

7.3 Nuclear structure

Up until 1932 only two elementary particles were known: the proton (hydrogen nucleus) and the electron. The alpha particle (helium nucleus) was thought to be a composite particle. During the following decade a number of other important particles were observed and our present theory of nuclear structure was established. This illustrates the fact that progress in scientific knowledge occurs in fits and starts, rather than smoothly.

In 1930 Bothe and Becker noticed that a very penetrating radiation was emitted when light elements, such as lithium or beryllium, were bombarded with alpha particles. Later experiments showed that this radiation must consist of either neutral particles or electromagnetic waves, since it was unaffected by an electric or magnetic field. The nature of this new radiation was explained by the experiments of Sir James Chadwick in 1932. When a nucleus of beryllium, for instance, is hit by an alpha particle, a neutral particle of about the mass of a proton is emitted. This particle is now called the *neutron*. Since the neutron has no charge and a mass of about one unit, its symbol is ${}_{0}n^{1}$.

Also in 1932, Carl D. Anderson discovered a new particle in *cosmic rays* (radiations coming from outside our solar system). This particle had all of the properties of an electron, except that its charge was positive instead of negative. This particle was called the *positron* and given the symbol ${}_{1}e^{0}$. In measurements on cosmic rays again in 1937–38 Anderson discovered another par-

ticle which carried either a positive or negative charge of one unit, but which had a mass more than 200 times the mass of an electron or a positron. This particle is called a *meson* (it is now more properly called a mu-meson, since other mesons have also been observed). Both the positron and the mesons have very short lifetimes.

In the formulation of any theory of nuclear structure, we would start with the electron, proton, neutron, alpha particle, positron, and various mesons. Since the positron and all mesons are unstable, we can discard them as possibilities. Also, the alpha particle is the nucleus of helium, so that it is presumably a composite particle. This leaves us with the electron, proton, and neutron as possible building-blocks for nuclei. For theoretical reasons, an electron could not exist for long in the very small space occupied by a nucleus, which is about 10^{-14} meter, but would quickly be expelled. For this reason it is believed that nuclei are constructed from protons and neutrons. We will later describe how electrons and alpha particles can be emitted from nuclei.

As the reader will remember from Sec. 6.4 and later discussions, each atom has a fixed nuclear charge, which is positive, and an equal number of electrons travelling about the nucleus. Thus, if the atomic number of an atom is Z, we will begin making its nucleus by using Z protons. In order to make the atom as a whole electrically neutral, we must have Z electrons moving about the nucleus. In order to make the mass of the atom equal to its mass number A, we must then add $(A - Z)$ neutrons. This scheme works because both the proton and neutron have masses which are almost exactly unity, while the mass of an electron is negligible. Examples of nuclear and atomic structure will be given in the following paragraph.

Let us first consider the alpha particle and its parent atom, helium, which has atomic number 2 and mass number 4. In order to construct this atom, we use 2 protons in the nucleus and 2 electrons travelling about the nucleus. In order to provide the proper mass for the nucleus, we must also add 2 neutrons.

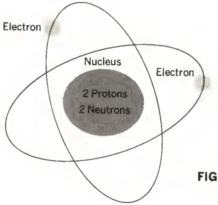


FIGURE 132

Schematically, an atom of helium consists of the particles shown in Fig 132. Similarly, the atom symbolized by ${}_{11}Na^{23}$ uses 11 protons and 12 neutrons in its nucleus and has 11 electrons travelling about the nucleus.

At this point the reader might wonder how protons, which have a positive charge, and neutrons, which have no charge, can exist stably in the small confines of the nucleus. Collectively, protons and neutrons are known as *nucleons.* At present the nature of the forces between nucleons is not known. However, we find experimentally that the masses of nuclei are always less than the sum of the masses of the protons and neutrons which make up a given nucleus. Thus, if we wish to break up a nucleus into its constituent particles, we must supply energy equivalent to the increase in mass needed. This energy is known as the *nuclear binding energy* of the nucleus in question. We see that although the nature of the forces between nucleons is not yet known, we can explain the stability of nuclei on the basis of energy considerations. Perhaps some day the forces involved in these energy changes will be known.

We must now explain the emission of electrons and alpha particles by nuclei, even though we do not believe that these particles are permanent residents of nuclei. It is believed that an alpha particle is formed inside the nucleus when 2 protons and 2 neutrons suddenly come together. This composite particle is immediately expelled by the nucleus and we observe an alpha particle. Similarly, inside of the nucleus a neutron may trans-

form itself into a proton and an electron according to the following equation:

$$ {}_0n^1 = {}_1H^1 + {}_{-1}e^0 $$

Once the electron is formed it is immediately emitted by the nucleus and we observe a beta ray.

If one or more electrons are removed from an atom, it then becomes an ion with a net positive charge. In this case the ion will experience both electric and magnetic forces when it moves through an electromagnetic field, as we discussed in Sections 4.1 and 4.3. By observing the effect of these fields on the moving ions, we can determine their mass.

A typical *mass spectrometer* is the one designed by Bainbridge and shown in Fig. 133. Only those ions which have a velocity such that the electric and magnetic forces just balance one another can pass through the velocity selector. The electrical force on an ion of charge Q in an electric field of intensity E is QE, according to Eq. 59. Similarly the magnetic force on the same moving with a velocity v at right angles to a magnetic field of intensity B is QvB. Thus, for an ion to pass through the velocity selector, the forces on it must satisfy the equation:

$$ QE = QvB \tag{106} $$

We see from Eq. 106 that only those ions with a velocity given by $v = E/B$ pass through the velocity selector. After leaving the

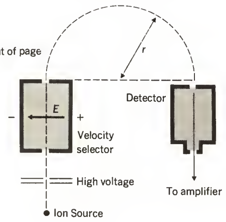


FIGURE 133

velocity selector, the ions experience only the magnetic force and move in circles of radius r according to the equation:

$$QvB = m\frac{v^2}{r} \tag{107}$$

When we combine Equations 106 and 107, the mass of the ions can be calculated from the measured values of the other quantities.

In 1912 Thomson found that some elements consisted of atoms with different masses. For instance, he found that neon had varieties with masses of 20 and 22. Otherwise, the various types of atoms of a given element are the same both physically and chemically. Two or more types of the same element which differ only in their masses are called *isotopes*. Since the chemical properties of an atom are determined by the number of electrons it has, all isotopes of an element have the same number of electrons travelling about their nuclei. In order to maintain electrical neutrality, all isotopes of a given element therefore have the same number of protons inside of their nuclei. We account for the differences in the masses of isotopes by adding additional neutrons to their nuclei, which provide the additional mass without adding charge. Oxygen occurs in 3 isotopes with masses 16, 17, and 18. Since the atomic number of oxygen is 8, an atom of each isotope has 8 electrons and 8 protons. However, ${}_8O^{16}$ has 8 neutrons, ${}_8O^{17}$ has 9 neutrons, and ${}_8O^{18}$ has 10 neutrons. Thus, isotopes of a given element have the same numbers of electrons and protons, but differ in the number of neutrons they have.

Two atoms may be the same in some other property aside from atomic number. When atoms have the same mass number, we say that they are *isobars*. A good example of some isobars is the triplet ${}_{18}A^{40}$, ${}_{19}K^{40}$, and ${}_{20}Ca^{40}$. If we call protons and neutrons collectively *nucleons*, isobars all have the same number of nucleons. Since isobars have different numbers of protons and electrons, they are different chemically. Even atoms which have both the same atomic number and the same mass number may differ in the emissions from their nuclei. These are called *isomers*. Naturally, isomers of a given element are otherwise physically and chemically identical. For instance, ${}_{21}Sc^{44}$ has two

isomers, one of which emits an electron with a half-life of 2.44 days and the other, a positron with a half-life of 3.92 hours. The reason for this odd behavior is not known at present. Here it should be stated that the theory of nuclear processes is far behind the experimental observations, so that we do not understand why nuclei with the same structure should emit different particles and have different properties.

7.4 Nuclear reactions

During the early part of this century many experiments were done using alpha particles as projectiles. Perhaps the most important of these experiments was the one in which Rutherford established the nuclear model of the atom, as was discussed in Sec. 6.4. In one of these experiments Marsden in 1914 noticed that when alpha particles bombarded hydrogen, particles identical to protons appeared. He interpreted his observation to mean that an alpha particle broke up a hydrogen molecule (which consists of two hydrogen atoms) into two parts, each one acting like a proton.

Rutherford repeated Marsden's experiment in 1919, using alpha particles to bombard such gases as air, carbon dioxide, and nitrogen, using apparatus as shown in Fig. 134. Protons were observed when air or nitrogen was the target, but not with carbon dioxide. Furthermore, protons were still observed even when all traces of hydrogen were removed. He therefore concluded that

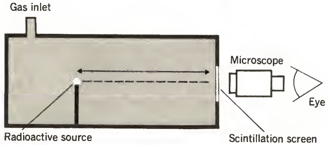


FIGURE 134

the protons were due to the presence of nitrogen. In order to explain the production of protons when nitrogen is bombarded with alpha particles, Rutherford hesitantly suggested that alpha particles converted nitrogen into an isotope of oxygen and liberated protons. The reaction he postulated is as follows:

$$_7N^{14} + {}_2He^4 = {}_1H^1 + {}_8O^{17}$$

In the reaction above he applied the rule discovered by Soddy and himself to explain the transmutations of the naturally radioactive elements. As we discussed in Sec. 7.2, nuclear charge and mass number must be conserved in any nuclear reaction, regardless of the number of particles involved. In this experiment the alchemists' dream of transmutation, or changing one element into another, had finally been accomplished. It should be noted, however, that the amount of the oxygen isotope produced was infinitesimal, since the chance of an alpha particle making a direct hit on a nitrogen nucleus is very small. Soon it was found that when other light nuclei are bombarded with alpha particles, protons are also produced in the transmutation.

We will now return to Chadwick's discovery of the neutron, which we described in Sec. 7.3. In this case beryllium was bombarded with alpha particles and neutrons were observed. For this reaction we can write:

$$_2He^4 + {}_4Be^9 = {}_0n^1 + {}_6C^{12}$$

As before, the amount of carbon produced is too small to measure, so we deduce it by knowing the other three parts of the reaction and using the Rutherford-Soddy rule. This is quite often the case in nuclear transmutations.

The nuclear transmutations we have discussed so far produced very small quantities of a stable isotope, such as $_8O^{17}$ or $_6C^{12}$, which could not be detected directly. The product of the reaction had to be deduced from a knowledge of the other particles involved and the use of the Rutherford-Soddy rule. In 1934 Joliot and Curie bombarded boron with alpha particles. Even

after the alpha-particle source had been taken away, positrons were emitted by the target with a half-life of about 10 minutes. They concluded that boron had been changed into a previously unknown isotope which was radioactive and emitted positrons. Furthermore, the positron activity obeyed the chemistry of nitrogen when the boron in the target was separated from the (presumed) new isotope of nitrogen formed in the target. They suggested that an isotope of nitrogen was produced in the following reaction:

$$_5B^{10} + {}_2He^4 = {}_0n^1 + {}_7N^{13}$$

The isotope of nitrogen postulated in the equation above is not known in nature, so they assumed that it emitted positrons according to the equation below:

$$_7N^{13} = {}_1e^0 + {}_6C^{13}$$

Later they found that when aluminum is bombarded with alpha particles, a similar positron activity is induced.

This discovery of *induced radioactivity* was tremendously important, since the product of a reaction could be identified by the chemical reactions it followed. Once the product of a reaction was known, its presence could be followed by observing the characteristic properties of its radioactivity: the type of particle emitted, the half-life of the radiation, and the energy of the particle. Thus, it was possible to know all four parts of a typical nuclear reaction. In addition, it is possible by this means to produce *radioisotopes* of elements which do not normally show radioactivity. Minute traces of these radioisotopes can be noticed by observing the individual particles emitted.

A few remarks on nuclear reactions are now in order. In the first place, the Rutherford-Soddy rule of conserving nuclear mass and charge is always obeyed. Also, more than one result may occur when a given target is bombarded with a certain projectile. Let us consider the bombardment of aluminum with *deuterons,* which are the nuclei of the rare isotope of hydrogen

of mass number two (${}_1H^2$). In this case we find experimentally that the following three reactions occur:

$$
{}_1H^2 + {}_{13}Al^{27} \rightarrow \begin{cases} {}_2He^4 + {}_{12}Mg^{25} & \text{(stable)} \\ {}_1H^1 + {}_{13}Al^{28} & \text{(radioactive)} \\ {}_0n^1 + {}_{14}Si^{28} & \text{(stable)} \end{cases}
$$

It should be noticed that all three of the reactions above obey the Rutherford-Soddy rule. The probabilities of the three reactions occurring depend on the kinetic energy of the incident deuteron, so that it is possible to favor one reaction by choosing the energy of the deuteron.

The neutron is particularly effective in producing transmutations. It has no charge and therefore is not repelled by the nuclear charge. It might appear that the electron, which is attracted to the positive nucleus, would be very effective in producing nuclear reactions. This is not the case, since only a small amount of energy transfer can take place between a very light particle and a heavy particle, such as a nucleus and an electron. (Consider, as an example, interactions between a tennis ball and a car.) Particles such as the proton (${}_1H^1$), deuteron (${}_1H^2$), and alpha particle (${}_2He^4$) are widely used as bombarding projectiles since their charges allow them to be accelerated to high energies in a variety of ways. Gamma rays (photons) occur when the energy of a nucleus changes from an excited energy state to a lower energy state. Since photons carry neither mass nor charge, they play a minor part in nuclear reactions. To sum up, nearly all nuclear reactions involve the impact of a heavy projectile on a target nucleus, with the products being another heavy particle and a slightly different nucleus. Reactions which involve electrons or photons as projectiles are relatively rare.

In a given nuclear reaction the sum of the masses of the bombarding particle and target nucleus may be greater than the sum of the masses of the product particles. According to the theory of

relativity discussed in Sec. 6.8, when an amount of mass, m, disappears we observe in its place an amount of energy, E, given by:

$$E = mc^2 \tag{108}$$

In Eq. 108 c is the velocity of light, which is 3×10^8 m/sec in the system of units used in this book. Thus, the loss of a very small amount of mass results in the appearance of a large amount of energy. A reaction in which mass is converted into energy is called *exoergic.* Similarly, the products of a nuclear reaction may have more mass than the projectile and target. In this case the reaction will not take place at all unless the projectile carries enough kinetic energy to furnish this additional mass as given by Eq. 108. Such reactions are called *endoergic.* From the above we see that the energy of the products of a nuclear reaction may be considerably greater or less than the energy of the projectile which caused the reaction. Measurements of these energies furnish the best proof of the theory of relativity, since otherwise observed changes in energy could not be explained.

Starting about 1930, a variety of machines were built in order to accelerate charged particles to high energies. Some of the best known of these are the cyclotron, Van de Graaff generator, betatron, and synchrotron. The operation of these machines will not be described here, but details can be found in any book dealing with nuclear physics. The reason for building these machines was to have available ions of high energy with which to bombard target nuclei and possibly produce transmutations.

In a typical bombardment reaction, boron is bombarded by deuterons (nuclei of ${}_1H^2$)and protons are produced. Using the Rutherford-Soddy rule for nuclear transmutations, we deduce that the following reaction must have occurred:

$${}_1H^2 + {}_5B^{10} = {}_1H^1 + {}_5B^{11}$$

(The transmuted nucleus could not have been ${}_5B^{11}$, as the product would then have been ${}_5B^{12}$, which is radioactive, but no radioactivity is observed experimentally.) If we look up the

masses of the particles appearing on the two sides of the nuclear equation, we can write:

$$
\begin{array}{llll}
{}_5B^{10} = & 10.016119 & {}_5B^{11} = & 11.012790 \\
{}_1H^2 = & \underline{2.014740} & {}_1H^1 = & \underline{1.008145} \\
 & 12.030589 & & 12.020940
\end{array}
$$

In this nuclear reaction, there is a loss in mass of 0.009919 atomic mass units, so this is an exoergic reaction. Thus, this reaction is energetically possible even when the incident deuterons have quite low energies.

7.5 Nuclear fission and fusion

After the discovery of the neutron by Chadwick in 1932, it was realized that this particle should be particularly effective in producing nuclear transmutations, since it would not be repelled by the nuclear charge. Enrico Fermi and his co-workers bombarded many elements with neutrons, and observed a large number of induced radioactivities. In a typical reaction of this kind, the product nucleus is unstable and emits an electron, as shown in the example below:

$$ {}_{11}Na^{23} + {}_0n^1 = {}_{11}Na^{24} $$

$$ {}_{11}Na^{24} = {}_{-1}e^0 + {}_{12}Mg^{24} $$

In the example above, we see that the net result is the production of an isotope of the element with an atomic number one unit greater than the atomic number of the target element.

When elements such as uranium and thorium were bombarded with neutrons, electron activities were produced which were assumed to be similar to the example given in the preceding paragraph. In the case of uranium, we would have then:

$$ {}_0n^1 + {}_{92}U^{238} = {}_{92}U^{239}; \qquad {}_{92}U^{239} = {}_{-1}e^0 + {}_{93}?^{239} $$

Since uranium has the highest atomic number of any element occurring in nature, these new elements were named *transuranic*

elements. Unfortunately, the number of activities induced by the bombardment of the heavy elements required the postulation of a large number of transuranic elements. Furthermore, no plausible decay scheme could account for all of the observed activities as due to a small number of transuranic elements. Thus, a dilemma existed in 1938.

During 1938 Hahn and Strassman found an activity which followed the chemistry of barium (atomic number 56) after uranium had been bombarded with neutrons. Exhaustive chemical tests convinced them that the activity was indeed due to an isotope of barium. To explain how barium was produced when uranium was bombarded with neutrons, they hesitantly suggested that the uranium nucleus broke up into two roughly equal fragments, according to the following equation:

$$_{92}U^{238} + {}_0n^1 = {}_{56}Ba^? + {}_{36}Kr^? + \text{neutrons}$$

As confirmation of this idea, they observed that a known radioisotope of krypton (Kr) accompanied the barium activity. This fragmentation of the uranium nucleus was named *nuclear fission* by Meitner and Frisch. Soon it was found that other heavy elements also underwent fission, such as thorium (atomic number 90) and proto-actinium (atomic number 91).

Very quickly, two important facts about fission reactions were learned. The sum of the masses of the fission products (Ba and Kr in the example above) is less than the sum of the masses of the neutron and the uranium nucleus. Thus, mass is converted into energy by such a process. Also, the amount of energy released per particle is about 100 times the amount released in ordinary nuclear reactions and about a billion times greater than the energy released in usual chemical reactions. A new and exceedingly powerful energy source had been discovered, if a way could be found to produce and use this energy on a large scale.

In addition to the large release of energy in nuclear fission, it was found that more than two neutrons were emitted in each such fission process. A sustained release of energy through nuclear fission could be obtained if at least one neutron from each fission later itself produced a fission reaction. Suppose that we start

with some neutrons in a lump of uranium. Some of the neutrons from fission will diffuse out through the surface of the lump and will not themselves produce a fission. However, the larger we make the lump of uranium, the smaller (relatively) do we make the rate of loss through the surface as compared to the rate of production of new neutrons in the volume. The minimum size of a lump of uranium which will barely sustain a fission reaction is called the *critical size.* For uranium this critical size is a sphere of radius about 1 inch in diameter. Impurities in the material which absorb neutrons increase the critical size.

Natural uranium metal consists of about 99.3 percent U^{238} and only 0.7 percent U^{235}. Neutrons emitted during fission have energies of about 2×10^{-13} joules and are called *fast neutrons.* Fast neutrons will produce fission in either isotope of uranium. However, in most cases a neutron gives up its energy in a series of collisions with nearby atoms, without producing fission. Since nuclei of U^{238} atoms are much more abundant (140 to 1), these slowed-down neutrons are absorbed and never produce fission. Only when the energy of the neutron is reduced to about 5×10^{-21} joules or lower does this absorption stop. At these low energies only the rare isotope, U^{235}, absorbs neutrons, and fission is produced. An isotope which undergoes fission with neutrons of any energy, such as U^{235}, is called *fissionable.* The heavier and more plentiful isotope of uranium absorbs so many neutrons without producing fission that a sustained nuclear fission reaction can not take place in natural uranium.

An obvious solution to the problem described in the preceding paragraph is to remove the heavier isotope and use only the light isotope. Since isotopes are chemically the same, any separation process must use the small difference between their atomic masses, which are 235 and 238 or about 1 percent. Various schemes were tried and all were successful to a greater or lesser extent. By 1945 fair sized amounts of nearly pure U^{235} became available, while today undoubtedly amounts measured in tons are stockpiled.

The inspiration for producing fissionable materials was the hope of making a super explosive which would put an abrupt end

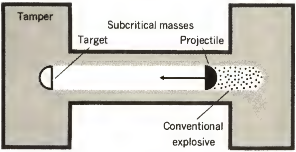


FIGURE 135

to World War II. In order to obtain a lot of energy quickly we need to use an amount of fissionable material much greater than the critical size. In this case the number of reactions would grow rapidly, rather than merely maintain a constant level. However, this much material could easily be triggered by a single stray neutron (perhaps from cosmic rays) and explode spontaneously. The solution to this problem is to keep apart two or more pieces of fissionable material, each of which is smaller than the critical size, and then to bring them together when an explosion is desired. A possible way of doing this is shown in Fig. 135. All difficulties were solved and the first two *atomic bombs* were dropped on Japan in August, 1945. The energy released by these first two bombs was about the same as that given off when 20,000 tons of TNT explode. Thus, the *nominal atomic bomb* is said to be a 20-kiloton bomb. Atomic bombs of both smaller and larger energy yields have since been tested.

Let us now return to the process in which a nucleus of U^{238} absorbs a neutron, producing U^{239}. This new isotope is unstable and decays by emitting an electron (beta particle). The result is a nucleus of element 93, now known as Neptunium (Np). Neptunium is also unstable and emits an electron, thus producing a nucleus of element 94, which is called Plutonium (Pu). The entire sequence is shown below.

$$ {}_0n^1 + {}_{92}U^{238} = {}_{92}U^{239} $$

$$ {}_{92}U^{239} = {}_{-1}e^0 + {}_{93}Np^{239} $$

$$ {}_{93}Np^{239} = {}_{-1}e^0 + {}_{94}Pu^{239} $$

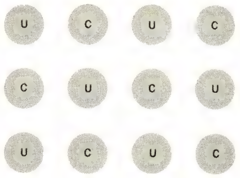


FIGURE 136

Plutonium was produced in large quantities by using some of the extra neutrons from U^{235} in a device known as a *nuclear pile reactor*, shown in Fig. 136, which is essentially a controlled atomic bomb. The reason for making this previously unknown element is that it is fissionable with neutrons of any energy and can thus take the place of U^{235} in an atomic bomb. The atomic bombs dropped on Japan were of different types. The one dropped on Hiroshima used U^{235}, while plutonium was used in the bomb dropped on Nagasaki.

One advantage of plutonium as compared with U^{235} is that it is prepared from abundant U^{238}. Also, it releases somewhat more energy in fission than U^{235}. Since the plutonium isotope has the long half-life of 25,000 years, once a certain amount has been produced it is permanent in terms of a man's lifetime and can be kept until there is a need for it.

Our sun is only an average star, but the energy it radiates is enormous. For instance, a square meter at right angles to sunlight receives energy at the rate of 1.3 kilowatts, and the total radiation from the sun is about 1.2×10^{34} joules per year. Since no changes in the sun's temperature have been observed over many years, calculations show that the sun is not merely a hot body cooling down. Thus, some other process must be suggested to account for the very large amount of energy emitted by the sun.

In 1938 Bethe and Weiszacker independently suggested two schemes which could explain the source of the sun's energy,

assumed to be produced by the conversion of mass. In each of their reactions the net result was the combination of four protons (hydrogen nuclei) to form a helium nucleus. This sort of process is known as *nuclear fusion*. Since the masses of the four protons are greater than the mass of the helium nucleus, energy is released. However, there is no need to worry about our sun, because it could continue radiating at its present rate for 8×10^{12} years and still use up only half of its mass. Since we know from spectroscopic and other evidence that hydrogen and helium are by far the most common elements in stars, these two processes seem very likely as the source of the energy of stars.

Hydrogen is also very plentiful on earth, mainly in water (H_2O). It would seem attractive to use a nuclear fusion process as a way to produce energy. Unfortunately, the probability of such a reaction taking place is very low, unless the hydrogen is subjected to very high pressure and temperature, such as is found in the interior of a star. The only equivalent environment available to man is the inside of an atomic bomb during the short duration of its explosion, although the required conditions might be produced as shown in Fig. 137. Various fusion reactions which take place under such conditions are shown below:

$$_1H^2 + {}_1H^2 = {}_0n^1 + {}_2He^3$$

$$_1H^2 + {}_1H^3 = {}_0n^1 + {}_2He^4$$

$$_1H^3 + {}_1H^3 = {}_0n^1 + {}_0n^1 + {}_2He^4$$

It should be noted that all of the reactions above produce an energetic neutron, which can contribute to the fission of the uranium or plutonium in the atomic bomb used as the trigger for the fusion reactions.

FIGURE 137

Using one or more of the reactions shown above, it should be possible to make a *fusion* or *hydrogen bomb*. This is not as easy or cheap as it sounds. *Deuterium* (${}_1H^2$) occurs in natural hydrogen with an abundance of only about 0.02 percent. *Tritium* (${}_1H^3$) does not occur in nature and must be produced by bombarding lithium with neutrons from expensive fissionable material. Furthermore, tritium has the short half-life of 12 years, so that its value quickly decreases if it is not used. Since the rate of a reaction depends on the concentration of the ingredients, both of these gases would have to be liquefied or even solidified in order to get a high concentration. This would complicate the mechanism of such a hydrogen bomb very considerably. On the other hand, there is no critical size for a fusion bomb, as the reactants are entirely inert until they are exposed to high temperatures and pressures. Fusion bombs using this or similar ideas have been tested. They have released energies of approximately 50 to 100 megatons of TNT equivalent, which is about 1000 times larger than the energy of an atomic bomb.

In order to avoid the complications involved in using only isotopes of hydrogen, other fusion reactions should be considered. Lithium hydride is a solid up to fairly high temperatures. The following fusion reactions release a good deal of energy:

$$
\begin{aligned}
{}_1H^2 + {}_3Li^6 &= {}_0n^1 + {}_4Be^7 \\
{}_1H^2 + {}_3Li^6 &= {}_2He^4 + {}_2He^4 \\
{}_1H^2 + {}_3Li^7 &= {}_0n^1 + {}_4Be^8
\end{aligned}
$$

In the reactions above it should be noted that both isotopes of lithium occur in nature and the hydrogen isotope is stable. Further, the nuclei which are to undergo fusion are already in close proximity in a molecule of solid lithium deuteride, so there is no problem with the concentration of the reacting nuclei. It seems likely that transportable fusion bombs use one or more of the reactions listed in this paragraph.

The energies of particles due to high temperatures are relatively low on the average. For instance, the average energy of a particle in equilibrium at a temperature of 10 million degrees

Celsius is about 2×10^{-16} joules. A singly charged particle accelerated through a potential difference of 1300 volts would gain the same energy. In this way we can give particles energies equivalent to the very high temperatures believed to exist in the interior of the sun. By means of this method, it is not hard to produce fusion reactions in the laboratory. However, it has been impossible so far to produce such reactions in large enough quantities for the energy output of the device to exceed its energy input. Research is now under way using highly ionized gases or *plasmas* accelerated and contained by electric and magnetic fields. If this effort is successful, fusion reactions, using the deuterium in sea-water, would be ample to supply man's energy needs for thousands of years to come. By contrast, it has been estimated that we will use up the fissionable materials on earth within a few centuries, while the fossil fuels, such as coal and oil, will be exhausted even sooner.

*7.6 Applications of nuclear physics

Although radium and a few other naturally radioactive elements were used in the treatment of cancer more than half a century ago, applications of nuclear physics were limited until induced radioactivity was discovered in 1934. Today radioactive isotopes of every element can be prepared. Since detectors of nuclear radiation can count individual nuclear disintegrations, extremely minute quantities of radioactive chemicals can be followed through various processes. The identity of each radioactive chemical can be determined by observing its characteristic radiation, energy, or half-life. It is easily seen that this method involving individual atoms is far more sensitive than the usual chemical techniques.

Suppose that we are interested in measuring the absorption by plants of phosphate in fertilizer. If we mix a small amount of radioactive phosphorus in with stable phosphorus, we can then follow the progress of these *tagged atoms* by observing their

radioactivity. It happens that there is an isotope of phosphorus with a half-life of about two weeks, which is just about right in terms of the growing season of a plant. Very useful research in agriculture has been done using this idea.

Another interesting use of tagged atoms or *radioactive tracers* is in the testing of the wear of piston-rings in an automobile engine. A radioisotope of cobalt is used in making the piston rings. The metal that is worn off during use falls into the crank-case oil. Measurement of the radioactivity appearing in the oil then allows us to calculate the minute amount of metal which has been worn off the piston-ring. Because of the extremely small amount of material involved, no ordinary chemical or physical measurement can give the information desired.

A few neutrons produced by cosmic rays continually change some of the nitrogen in the atmosphere into radioactive carbon, the reaction being as follows:

$${}_7N^{14} + {}_0n^1 = {}_1H^1 + {}_6C^{14}$$

While they are alive, plants and animals absorb both stable and radioactive carbon in the proportions of about 10^{12} to 1. Once an organism dies the radioactive carbon is not replenished and decays into stable nitrogen. Since the half-life of ${}_6C^{14}$ is 5720 years, the amount of radioactive carbon drops to half of its value 5720 years after the organism dies. If we compare the ratio of stable to radioactive carbon in a specimen to the ratio today, we can determine the age of the specimen. Using this *radiocarbon dating* method, Libby and others have measured ages of objects as far back as 25,000 years.

Medical applications of radioisotopes are numerous. Many such applications merely use a radioactive material as a source of radiation to cure a cancer, which was first done using radium or x rays. More interesting uses take advantage of the fact that certain elements tend to concentrate in certain organs or parts of the body. For instance, iodine is mainly absorbed by the thyroid gland. If a patient takes a dose of radioactive iodine, it will settle in his thyroid and may destroy a cancer there without

hurting much other tissue. Other elements are used to treat cancers in other parts of the body.

As a final example of the peaceful use of radioactivity, we will discuss the production of power from fissionable materials. In a nuclear pile reactor producing plutonium, approximately 22,000 kwh of energy are liberated for each gram of plutonium made. In other terms, the production of one million kilowatts of power continuously for a year would only require the consumption of about two tons of fissionable material. This large amount of energy from a small amount of fuel is at present the chief advantage of nuclear power. A nuclear-powered submarine, for instance, can stay at sea almost indefinitely. Nuclear power has obvious advantages in locations where other fuels are scarce or expensive. At present, however, the cost of electrical energy generated by nuclear means is more than that of energy generated using conventional methods. As technology develops and as our reserve of fossil fuels decreases, it can be expected that nuclear power will become more economical.

Before closing this section, a word about the health and safety aspects of radioisotopes would seem to be in order. Charged particles, such as electrons and alpha particles, are absorbed by the skin. Thus, they do very little internal damage but may cause very severe burns or cancers. Gamma rays penetrate deeply into the body; their effects were described earlier in Sec. 6.6. Neutrons are also very penetrating and do damage by knocking protons out of the complex molecules which make up cells and tissue. To a good approximation the roentgen unit defined in Sec. 6.6 measures the damage done by gamma rays and neutrons per unit of volume exposed. From experience, a dose over the whole body of perhaps 10 roentgens per year is tolerated by the body, although there is no lower limit for a safe radiation dose. The dose due to cosmic radiation is of the order of 10^{-2} roentgens per year, but even this small amount of radiation is undoubtedly responsible for numerous mutations every year. Certainly any unnecessary exposure to radiation should be avoided. On the other hand, the dangers of radiation should not prevent it being used when the benefits are likely to be great.

*7.7 Cosmic rays and elementary particles

In 1903 Rutherford and Cooke found that about ten ions were formed per second in a cubic centimeter of air at sea level. They concluded that this ionization was due to small amounts of radioactive elements near their instruments. In 1911 Hess showed that the amount of ionization rose rapidly when he carried his instruments up to heights of several miles using balloons. For instance, at a height of three miles the rate of ionization was about three times its value at sea level. Some recent data are shown in Fig. 138. Since the instruments were far from sources of radiation on the earth, Hess suggested that the ionization must be caused by radiation from outer space, the *cosmos*. Thus, he named the unknown source of this ionization *cosmic rays*. Because the ionization intensity varied only slightly during various times of the day or seasons of the year, he further concluded that our sun was not chiefly responsible.

Cosmic rays are detected in the same way as nuclear radiations, as was described in Sec. 7.1. Since the earth has a magnetic field, it will deflect positively charged particles in the opposite direction to negatively charged particles. Also, a magnetic field has no effect on neutral particles or electromagnetic waves. Observations show that cosmic rays must be predominately positively charged particles, since these radia-

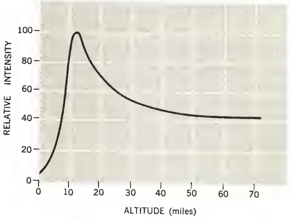


FIGURE 138

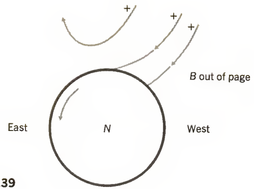


FIGURE 139

tions arrive mainly from the west as shown in Fig. 139. Furthermore, the magnetic field of the earth has more effect on particles arriving near the equator than on particles arriving near the poles of the earth, thus lending additional support to the idea that cosmic rays consist of charged particles. More detailed observations show that cosmic rays consist of approximately 77 percent hydrogen ions (protons), 20 percent helium ions, and 3 percent other positive ions. It is interesting to note that these proportions are almost the same as the proportions of elements in stars as determined from their spectra.

When the cosmic radiation is examined a few miles from the earth's surface, it is found that there is a component, called the *soft component,* which is easily absorbed and another component which is very penetrating and is called the *hard component.* Furthermore, the atmosphere alone is sufficient to absorb the soft component before it reaches sea level. From this we conclude that the soft component cannot be the primary cosmic radiation. Instead, the soft component must be created at all levels in the atmosphere by the hard component. Thus, the hard component must be the primary cosmic radiation.

Let us suppose that the primary cosmic radiation consists of very energetic positively charged particles. If such a particle has a collision with a nucleus in the atmosphere, the sudden deceleration of the cosmic-ray particle will lead to the emission of electromagnetic radiation. This is similar to one of the

processes by which x rays are emitted, as described in Sec. 6.6. These photons can then produce ionization of atoms and molecules they strike. Also, the primary cosmic-ray particle may produce transmutations in nuclei it strikes, again producing ionizing particles. If the energy of a primary cosmic-ray particle is high enough, it may engage in many such processes as it passes through the atmosphere. This secondary ionization produced by the primary cosmic-ray particles is then what we called above the soft component of cosmic radiation. The soft component is therefore not from the cosmos at all, but is produced by energetic cosmic-ray particles passing through our atmosphere.

Some of the primary cosmic-ray particles have enormous energies. Many remarkable nuclear reactions were first observed as the result of cosmic-ray bombardment, as we discussed in Sec. 7.3, although during the last ten years man has been able to duplicate them by accelerating protons to equivalent energies. The electron-like positively charged particle (positron) observed by Anderson in 1932 has a very short lifetime and is an example of an *anti-particle,* the corresponding particle in this case being the electron. In the past few years the anti-proton and the anti-neutron have been observed, as man has been able to reproduce in the laboratory the exceedingly high energies formerly only available in cosmic rays. We might even speculate that there is an anti-universe made up of anti-particles in combinations similar to those we know using particles.

It is only possible to produce a particle and its corresponding anti-particle simultaneously as a pair. For instance, an electron-positron pair may be created by a sufficiently energetic x ray, as is shown in Fig. 140. In order for momentum to be conserved, the electron and positron travel off with equal energies in directions making equal angles with the original direction of the x ray, as is shown in the diagram. This process is known as *pair-formation.* Here we have an example of the transformation of the energy hf of the x-ray photon partly into the masses of the two particles and partly into the kinetic energies the particles acquire.

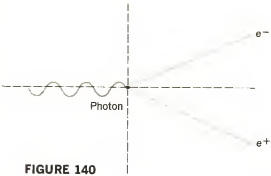


FIGURE 140

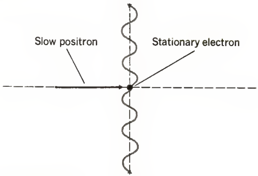


FIGURE 141

After the positron has been slowed down by collisions with atoms, it may encounter a slowly moving electron. In this case the two particles combine or coalesce, and all of their mass is converted into energy in the form of radiation. This process is known as *pair-annihilation*. Since the positron and the electron are moving very slowly before they combine, their momentum is very nearly zero. In order to have no net momentum after they combine, pair-annihilation leads to the emission of two photons of equal energy travelling in opposite directions. These ideas are illustrated by Fig. 141. If the mass of either particle is called m, the frequency of pair-annihilation radiation is seen to be given by the equation:

$$mc^2 = hf \tag{109}$$

Thus, a photon produced in pair-annihilation can be readily identified by its characteristic frequency. Here it should be noted that the remarks of this paragraph and the one preceding also apply to interactions between protons and anti-protons or other particles and their anti-particles.

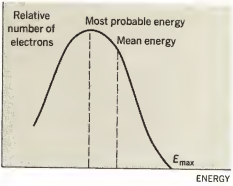


FIGURE 142

As we said earlier, a new particle called the *meson* and having a mass intermediate between those of the electron and the proton was discovered in 1937. Later other particles were observed which also had masses of approximately the same value. Furthermore, the various types of mesons occurred with positive, negative, or zero charge. The three varieties of mesons have been labelled mu-mesons, pi-mesons, and K-mesons. Some of their properties are summarized in a table later in this section.

After the discovery of the various mesons, additional particles were found to be produced by cosmic rays. Since these particles all had masses slightly greater than the proton mass, they were called *hyperons*. Even more recently man has been able to use high energy accelerators to produce these particles in the laboratory. The exact part that hyperons and mesons play in nuclear physics is not known as yet, but controlled experiments with high energy accelerators may provide the answer.

When the energies of particles emitted by unstable nuclei are measured, it is usually found that the particles have a very definite energy or possibly one of several definite energies. An exception to this statement is found when we measure the energies of electrons emitted by nuclei. In this case we find that the energies of the electrons have a wide and continuous distribution. The typical distribution of the energies of the electrons emitted by a nucleus is shown in Fig. 142. As we see in

the diagram, the most likely energy of an electron is about half the maximum energy of electrons emitted by this particular nucleus. Furthermore, there is a definite upper limit to the energy of a given electron, which is labelled E_{max}.

If we assume from our knowledge of other nuclear radiations that a given nucleus has available a certain amount of energy, we would expect that all electrons from a given type of nucleus would have the same energy. This is not found to be the case, since electrons are emitted with variable amounts of energy. In addition, it is also found experimentally that the momentum of the electron is not equal to the momentum of the recoiling nucleus. Thus, both energy and momentum do not appear to be conserved when a nucleus emits an electron.

We could solve the dilemma presented in the preceding paragraph simply by assuming that in this process energy and momentum are not conserved. To most physicists, however, this is a most unattractive idea. In 1931 Wolfgang Pauli proposed an alternative. According to his hypothesis, a previously unknown particle could carry away the excess energy and momentum. This particle is called the *neutrino* (little neutron), since it would have no electrical charge. Several years later Fermi used Pauli's idea by assuming an interaction between the nucleus, the electron, and the neutrino at the instant of emission of the electron and the neutrino. Fermi's theory correctly predicted the relative numbers of electrons emitted with various energies shown in Fig. 142.

Until recently, the success of Fermi's theory in accounting for the distribution in energy of electrons emitted by nuclei was the chief reason for believing in the neutrino. In 1956 Cowan and Reines reported the first experimental detection of the neutrino. Later experiments by them and others left no doubt of its actual existence. This elusive particle is believed to have no charge or rest mass, although it does have a dynamical mass corresponding to its energy according to the theory of relativity.

Let us define an *elementary particle* as one which is not a composite of other particles. At this date we feel quite sure that an alpha-particle is not elementary, since we believe that it

	Particle	Relative mass	Charge	Stability
Leptons	Photon	0	0	Yes
	Neutrino	0	0	Yes
	Electron	1	+, −	Yes
Mesons	Mu	207	+, −	No
	Pi	264	0	No
	Pi	273	+, −	No
	K	966	+, −	No
	K	974	0	No
Nucleons	Proton	1837	+, −	Yes
	Neutron	1839	0	Almost
Hyperons	Lambda	2185	0	No
	Sigma	2330	+	No
	Sigma	2336	0	No
	Sigma	2346	−	No
	Xi	2570	0	No
	Xi	2583	−	No
	Omega	3280	−	No

ELEMENTARY PARTICLES

consists of two protons and two neutrons. On the other hand, the proton itself is almost certainly an elementary particle. If we consider particles such as mesons and hyperons, we cannot be sure. The accompanying table shows those particles now considered elementary. The reader should here be warned that the list changes from year to year. It is the opinion of this author and many other physicists that most of the particles listed are actually composites or excited states of a small number of elementary particles. We hope further research will resolve this question.

After a lengthy digression, let us return to cosmic rays. Until recently these were man's only source of high energy particles. Even though cosmic rays could not be controlled by man, these energetic particles allowed us to discover the positron, various mesons, and various hyperons. In the future it is

likely that most of our knowledge of these particles will be obtained by using high energies produced on earth under laboratory conditions. Nevertheless, we should be glad that nature furnished us with a cosmic high energy machine before man could do the same himself.

No successful theory of the origin of cosmic-ray particles with very high energies has yet been proposed. For instance, weak electric fields extending through great distances in interstellar space could give charged particles huge energies. However, it has not been possible to explain how such electric fields would be maintained for long periods of time. Similarly, a changing magnetic field in a vast region of space could also accelerate charged particles to high energies. Again, the source of such a magnetic field remains to be explained. In the true spirit of research, we can only hope and believe that additional knowledge will lead to an explanation of the origin of this mysterious but most interesting radiation.

SUMMARY

Becquerel found that elements such as uranium emitted radiations which could affect photographic plates, ionize gases, and cause fluorescence in certain materials. This property he named radioactivity, and it was soon found by the Curies and others that a number of other elements also were radioactive. This discovery led to the whole field of nuclear physics, which is so important today.

When the nature of these radiations was studied more closely, it was found that there are three types of emissions. Alpha rays are doubly ionized helium ions, beta rays are electrons, and gamma rays are electromagnetic waves similar to x rays. The distinctive properties of these three radiations were then discussed, and the evidence that they come from nuclei was presented.

After a nucleus emits a particle, a new nucleus is produced in such a way that the total charge and mass of the various particles involved is conserved in the process. Thus, if we know the original nucleus and the particle emitted, we can predict the nature of the nucleus produced. This idea was later applied to nuclear transmutations produced by bombarding nuclei with high-energy particles. We saw also that various nuclei of elements disintegrate at slow or fast rates, so we defined the half-life for each type of disintegration as the time required for half of the nuclei to disintegrate.

The next problem we considered was the structure of nuclei. Here we considered the evidence for the idea that a nucleus consists of a number of protons sufficient to give the correct nuclear charge (atomic number) and an additional number of neutrons to provide the proper nuclear mass (mass number). This idea has been very successful in explaining the large variety of nuclei found in nature. Furthermore, we can then account for transmutations produced when nuclei are bombarded by various particles of high energy.

There is always a change in mass when a nuclear transmutation is produced. According to the theory of relativity, a proportional amount of energy is released if mass decreases or must be put in if the mass increases. This led us to discuss nuclear fusion and fission, in which the energy released may be very large indeed. This energy may be used quickly in a bomb or released slowly to provide energy for peaceful uses. The processes by which stars use nuclear fusion to produce vast amounts of energy were also discussed. Finally, a few of the numerous applications of nuclear physics were then discussed, and a brief account of cosmic rays was presented.

PROBLEMS

1 *Express the intensity of cosmic rays at sea level in roentgens per day.*

ANS.: *Approx.* 4×10^{-4}.

2 *Determine the product when* $_{86}Rn^{222}$ *decays by the emission of an alpha particle.*

3 *Determine the product when $_1H^3$ (tritium) decays by the emission of an electron.* ANS.: $_2He^3$.

4 *Determine the product when $_{11}Na^{22}$ (a radioisotope of sodium) decays by the emission of a positron.*

5 *Strontium-89 has a half-life of 55 days. If the initial activity of a sample is 5 millicuries, compute its activity at the end of 100 days.* ANS.: *1.3 millicuries.*

6 *Observations taken six days in a row on a sample of radon gave the following activities in particles per minute: 1560, 1304, 1090, 908, 760, and 634. From these measurements calculate the approximate half-life of radon.*

7 *State the numbers and approximate locations of the various particles which make up an atom of $_{10}Ne^{22}$.*
ANS.: *Nucleus contains 10 protons and 12 neutrons; 10 electrons move about the nucleus.*

8 *Repeat Problem 7 for $_{94}Pu^{239}$.*

9 *Suppose that you were given 15 electrons, 18 protons, and 16 neutrons. What is the most massive neutral atom you could construct using these materials?* ANS.: $_{15}S^{31}$.

10 *If you had 8 protons and 10 neutrons, what other particles would you need to construct a neutral atom?*

11 *When $_6C^{12}$ is bombarded with deuterons ($_1H^2$), protons are observed. What is the product of this reaction?* ANS.: $_6C^{13}$.

12 *When $_7N^{14}$ is bombarded with neutrons, $_6C^{14}$ (radioactive) is produced. What is the other particle involved in the reaction?*

13 *You hope to produce $_7N^{13}$ (which is radioactive) by bombarding $_8O^{16}$ with protons. What light product-particle would you expect to observe?* ANS.: *Alpha particle.*

14 *Suppose that the neutrons from each fission process produce on the average two fissions within a time of 10^{-6} sec. Compute approximately the time for a single initial fission to lead to a total of one thousand fissions.*

15 *Repeat Problem 14, but find the time for a single initial fission to be the forerunner of a million fissions.* ANS.: *10^{-4} sec.*

16 *Compute the change in the sun's mass per second, given that the sun radiates about 1.2×10^{34} joules per year in the form of radiant energy.*

17 *$_2He^6$ is unstable, emitting an electron. What is the (stable) product of the decay?* ANS.: $_3Li^6$.

18 *Suppose that a certain amount of tritium is produced for use in hydrogen bombs. If the half-life of tritium is 12 years, approximately what percentage loss would be incurred if this tritium were not used for 50 years?*

19 *Wood from the coffin of a pharaoh shows only half the radioactivity per gram that wood today shows. Approximately at what date did the pharaoh die?* ANS.: *Approx. 3760* B. C.; *See Sec. 7.6.*

20 *Explain why absorbing a small amount of radium (half-life 1600 years) is very dangerous, while doses of iodine-131 (half-life 8 days) are given routinely in hospitals to patients.*

21 *What is the product when $_{92}U^{235}$ emits an alpha particle?* ANS.: $_{90}Th^{231}$.

22 *What is the product when $_{11}Na^{24}$ emits an electron?*

23 *What is the product when $_{27}Co^{60}$ emits an electron and a gamma ray?* ANS.: $_{22}Ni^{60}$.

24 *What particle is emitted when $_{92}U^{238}$ transforms itself into $_{90}Th^{234}$?*

25 *If a sample of $_{14}P^{32}$ (half-life 14 days) emits 200 particles per second initially, how many particles per second will it emit 6 weeks later under the same conditions?* ANS.: *25.*

26 *The half-life of plutonium is about 25,000 years. If this material costs \$10,000 per lb to make, what would its value be 100 years from now, if no inflation occurs?*

27 *The half-life of tritium, which is used in hydrogen bombs, is about 12.5 years. If this material costs \$5000 per lb to make, what would its value be 50 years from now, if no inflation occurs?* ANS.: *\$313/lb.*

28 *State the numbers and approximate locations of the particles which make up an atom of $_{11}Na^{23}$.*

29 *Repeat Problem 27 for an atom of $_{92}U^{234}$.* ANS.: *92 protons and 142 neutrons in nucleus and 92 electrons in orbit about the nucleus.*

30 *What particle is emitted when* $_{82}Pb^{214}$ *transforms itself into* $_{83}Bi^{214}$*?*

31 *What particles are necessary to a construct a neutral atom of* $_{8}O^{18}$*?*
ANS.: *8 protons, 10 neutrons, and 8 electrons.*

32 *When* $_{5}B^{10}$ *is bombarded with deuterons, protons are observed. What nucleus is produced by this reaction?*

33 *When* $_{3}Li^{6}$ *is bombarded by slow neutrons, alpha particles and tritium nuclei are produced. Write the equation for this reaction.*
ANS.: $_{0}n^{1} + {}_{3}Li^{6} = {}_{2}He^{4} + {}_{1}H^{3}$.

34 *Suppose that you expect to produce* $_{6}C^{14}$ *by bombarding* $_{5}B^{11}$ *with alpha particles. What light product-particle would you expect to observe?*

35 *Suppose that a way is found to convert mass directly into energy for use in place of food. If your body requires 3000 kilocalories of energy per day, compute the mass which would have to be used up daily to supply this requirement.*
ANS.: 1.39×10^{-10} *kg.*

36 *Wood from an ancient campfire shows only* $\frac{1}{8}$ *the radioactivity that wood today shows. Approximately at what date was the fire made?*

37 *The mass of an atom (hypothetical) of atomic weight unity is called one* atomic mass unit *and its value is approximately* 1.66×10^{-27} *kg. Calculate the energy released when one atomic mass unit is converted entirely into energy.*
ANS.: 1.49×10^{-10} *joules.*

38 *The* electron-volt *is defined as the energy acquired by an electron when it passes through a potential difference of one volt. Since the charge of the electron is approximately* 1.60×10^{-19} *coulombs, one electron-volt =* $1.60 \times 10^{-19} \times 1 = 1.60 \times 10^{-19}$ *joules. Calculate the energy equivalent to one atomic mass unit in electron-volts.*

DISCUSSION QUESTIONS

1 *Discuss the element of luck in Becquerel's discovery of radioactivity.*

2 *Why is an ionization chamber used to measure the electrons knocked off atoms rather than to measure the number of ions produced?*

3 *If your body received a dosage of two milliroentgens per day, how long would it take for you to receive a total of one roentgen? How long would it take for you to receive a lethal dose of 600 roentgens? (This is about thirty times larger than the dosage due to cosmic rays).*

4 *Radium possessed by a hospital is not ordinarily used in treating patients. Instead, the radioactive gas radon given off by radium is sealed in platinum ampules and these are used to treat patients. What is the reason for this?*

5 *The half-life of* $_{14}P^{32}$ *is two weeks while that of an isotope of strontium found in the fall-out from a nuclear explosion is approximately half a year. Both elements settle in the bone-marrow and may cause leukemia. Why can the phosphorus isotope be used fairly safely, while the strontium isotope is quite dangerous?*

6 *Why is it easy to separate various isobars from one another, but difficult to separate isotopes?*

7 *Is there any way you can think of to separate isomers?*

8 *If you wanted to produce a certain radioactive isotope by bombarding a target with high energy particles, by what criteria would you be guided in your selection of the target material?*

9 *Large accelerators can produce many transmutations per second. By selecting an exoergic reaction, each transmutation releases energy. Why are such machines not used as energy sources?*

10 *Science fiction writers sometimes suggest that in the future cars will be fueled with a pinch of uranium at the factory and never again will have to be refueled. Discuss the difficulties involved.*

11 *Discuss the difficulties in powering an aircraft with nuclear fuel.*

12 *The half-life of uranium is about 4.5 billion years and its end product is lead, which is stable. Discuss how measurements on the relative abundances of uranium and lead in an ore sample could be used to estimate the age of the earth.*

13 *Explain how a patient can be safely given a dose of radioactive iodine to cure a thyroid cancer without danger of producing additional cancer by the radiation emitted by the iodine. (The half-life of this isotope is eight days.)*

14 *Polycythemia is the overproduction of red blood cells by the bone marrow. Suggest a scheme by which this disease could be arrested by the use of radioactive phosphorus.*

15 *Discuss a method by which a radioactive isotope could be used to measure the thickness of a steel plate passing through a rolling mill.*

CHAPTER EIGHT

FUTURE POSSIBILITIES

So far we have discussed facts and theories which are quite well established. In this concluding chapter we will talk about possibilities of the future. Almost certainly not all of the ideas presented will take place during the lifetimes of the readers of this book, but we almost guarantee that some of them will. In addition, scientific discoveries not now dreamed of will probably be made during the next few decades. Thus, scientific advances will be made in totally unexpected directions.

In electricity it is possible to counteract the effect of a positive charge by using a negative charge. In a similar way man may

find a way to counteract the force of gravitation by discovering *anti-gravity*. At present we find that all particles attract one another gravitationally, but perhaps gravitational repulsion will some day be produced. The numerous applications of such a principle are obvious.

As a more certain application of mechanics we can mention trips by men to the moon or the other planets. Such voyages will almost certainly take place during the next few years. Perhaps we will find that the moon is completely barren, but it is possible that there are important mineral deposits or other items of value. Similarly, until we explore planets such as Mars and Venus we will never know what they are really like. Explorations of this sort might change civilization as much as the discovery of America in 1492 did.

Trips to possible solar systems associated with other stars are much less likely in the near future. One of the nearest stars is Alpha Centauri, which is about four light-years away from our sun. (A *light-year* is the distance travelled by light in one year at the rate of 186,000 miles per second. It equals about six million million miles.) Since the speed of present satellites is about seven miles per second, it would take about 100,000 years to travel to this star and an equal time to return. Perhaps when rockets can be powered with nuclear fuel, high enough speeds can be reached so that a trip to a nearby star can be made during a man's lifetime. Then we will finally learn if there are planets and life associated with stars aside from our sun.

As we discussed in Sec. 1.1, a great deal of progress has been made during this century in explaining the facts of chemistry on a physical basis. In particular, wave or quantum mechanics accounts for the properties of atoms and molecules. It is possible that biology will also be explained in a similar way. Once we understand the nature of life, we can expect to be able to cure or prevent cancer and many other ailments. Already a great deal is known about the way in which cells duplicate themselves. Fairly soon it should be possible to make simple cells in the laboratory. Once we understand the physical basis of life, many things should become possible.

During the past few years a remarkable new optical device known as a *laser* has been developed which is like the maser discussed in Sec. 6.5. (The word *laser* is an acronym based on the phrase *L*ight *A*mplification by the *S*timulated *E*mission of *R*adiation.) A laser can produce a very intense and concentrated beam of light. It has already been used to heat small areas and to cut holes in metals. It will also be used to send signals over great distances, since the energy of the beam is not diffused as is the case with other sources of radiation. Because of the intensity of a laser beam, it might also turn out to be similar to the death ray of science fiction.

Many applications of nuclear physics are likely during the next few years. Nuclear power has already been used in submarines and ships. Research is going forward to develop a nuclear-powered aircraft, and it is probable that someday cars will also use nuclear power. Perhaps in the future you will buy a new car with enough fuel installed to drive the car during all of its useful life. If cooling and lubrication systems can be similarly improved, you might visit a service station only occasionally.

In many parts of the world fresh water is scarce, although salt water is plentiful. If we boil salt water and then condense the vapor, we obtain fresh water, since the salt does not go off with the vapor. However, the fresh water obtained in this way is costly, because of the expense of heating the salt water. Small plants using nuclear power have already produced fresh water, and we can expect that in the future additional applications of this process will be made.

As we mentioned in Sec. 7.5, a nominal or typical atomic bomb releases as much energy as the explosion of 20,000 tons of TNT. One obvious application of this enormous amount of energy is to move mountains or to dig canals. It is quite likely that a second canal connecting the Atlantic and Pacific Oceans will be built with the use of nuclear energy. In addition, hurricanes might be diverted or the polar icecaps might be changed. It is fortunate that the large amount of nuclear material produced for military purposes can also be used to help mankind.

When uranium or plutonium undergoes fission in a nuclear pile reactor and thus produces energy, the resulting fission products are intensely radioactive. At present there is a real problem in disposing of these by-products of nuclear reactors. Typically, they are buried or dropped into the sea in sealed cans. It has already been shown on a small scale that the radiations from these materials kill the bacteria which cause spoilage of food. In the near future it seems quite likely that food will be preserved by irradiation. Perhaps we will have radiation units in our homes which will make canning or freezing of food unnecessary.

Whenever man learns more about nature, practical uses of this knowledge usually follow quickly. As we discussed in Sec. 6.6, within three months after the discovery of x rays they were used by doctors in setting fractures. This important discovery came from Roentgen's study of the conduction of electricity through gases, which certainly might have seemed to be unrelated to medical research. Similarly, Einstein's work on relativity eventually led to the atomic bomb and the production of nuclear energy. We can say rather generally that all additions to our fundamental knowledge finally benefit mankind. At the time the research is going on, it is not possible to predict exactly the ways in which the new information will be used. This is typical of research, and it has been said that "if you know what you are going to find, you are not doing research." After this remark we can only conclude by saying that undoubtedly discoveries will be made which are not even imagined today.

DISCUSSION QUESTIONS

1 *Since the force of gravity on the moon is much less than on the earth, a man should be able to jump proportionately higher. Does this seem plausible to you?*

2 *Suppose that valuable mineral deposits were discovered on the moon. Do you think it would be economical to mine them and send them back to the earth?*

3 *It has been suggested that a way of reaching distant stars would be to send along entire families. Their descendants many generations later might then eventually return to the earth. Discuss the advantages and disadvantages of this plan.*

4 *Certain numbers, such as the ratio of the circumference of a circle to its diameter (π), presumably are the same everywhere in the universe. We know of radio waves from outer space already, and perhaps some day we will be able to translate some of the material we receive. Discuss whether or not we should answer such a communication, and if so, how.*

5 *Carbon is the most important element for living things on this planet. Silicon is chemically very similar to carbon. However, carbon dioxide is a gas while silicon dioxide is a solid (quartz). Discuss possible forms of life on a planet where silicon played the major role.*

6 *The lowest pressures now attainable still leave about a billion gas molecules in each cubic centimeter. It has been suggested that a tall, vertical pipe could reach to points where the remaining pressure would be even lower than we can now achieve. We would then have a "source" of vacuum. Discuss the feasibility of this idea.*

7 *Water is the most important liquid on our earth. Nearly all of the time it remains in the liquid state, and it is a very good solvent. Discuss the conditions which should exist for ammonia to be the most important liquid. (A whole field of chemistry has developed based on ammonia as the solvent).*

8 *The deeper we drill below the surface of the earth, the higher the temperature becomes. Discuss the feasibility of recovering this heat energy by drilling a very deep hole.*

9 *It has been suggested that nuclear energy could be used to melt much of the polar ice and thus make the climate warmer. Discuss the probable effects of this on our civilization.*

10 *During this century we have seen the development of radio, television, nuclear energy, and many new drugs, just to name a few discoveries. Do some brainstorming and try to guess what new discoveries will be made during the remainder of this century.*

INDEX

A

B

C

D

E